Excerpts from Kolmogorov's Diary

Translated by Fedor Duzhin

Introduction

At the age of 40, Andrey Nikolaevich Kolmogorov (1903–1987) began a diary. He wrote on the title page: "Dedicated to myself when I turn 80 with the wish of retaining enough sense by then, at least to be able to understand the notes of this 40-year-old self and to judge them sympathetically but strictly".

The notes from 1943–1945 were published in Russian in 2003 on the 100th anniversary of the birth of Kolmogorov as part of the opus *Kolmogorov* — a three-volume collection of articles about him.

The following are translations of a few selected records from the Kolmogorov's diary (Volume 3, pages 27, 28, 36, 95).

Sunday, 1 August 1943. New Moon

6:30 am. It is a little misty and yet a sunny morning. Pusya[1] and Oleg[2] have gone swimming while I stay home being not very well (though my condition is improving).

Anya[3] has to work today, so she will not come. I'm feeling annoyed and ill at ease because of that (for the second time our Sunday "readings" will be conducted without Anya).

Why begin this notebook now?

There are two reasonable explanations:

1) I have long been attracted to the idea of a diary as a disciplining force. To write down what has been done and what changes are needed in one's life and to control their implementation is by no means a new idea, but it's equally relevant whether one is 16 or 40 years old.

2) Now that I'm 40, I feel more deeply as life flows and goes by since past experience has an independent significance as compared to what one expects at 16 or even 30, when everything is viewed as a preparation for a distant future. Hence the need to capture the present at the very moment it transits from the non-existence of something that has yet to happen to the non-existence of something that has already passed by.

Possibly, the third reason put forward is contentious.

3) The period of "psychological research"[4] that began in February has been decided to be brought to an end. That caused a certain slackness in me and Pusya in July. So, this diary is being pursued to restore the discipline and at the same time to allow the passion for psychological research to be released in a more organised manner. Hopefully, there will not be too much of this research.

Eventually, this notebook might see some memories, thoughts, psychological analyses besides short current notes, but it will only happen after I have put my life in order.

Tasks to do now:

1) Finish small jobs as soon as possible (shooting,[5] quality control,[6] Lobachevsky[7]).

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[1] Pusya – nickname of Pavel Sergeyevich Alexandrov.
[2] Oleg – Oleg Sergeyevich Ivashev-Musatov, Kolmogorov's son-in-law.
[3] Anya – Anna Dmitriyevna Kolmogorova, Kolmogorov's wife and Oleg's mother.
[4] Kolmogorov's work *Pyotr Ilyich Tchaikovsky* written for the 50th anniversary of the composer's death and devoted to Pavel Alexandrov was never published.
[5] A collection of papers on the theory of shooting, edited by Kolmogorov, published in 1945.
[6] One of Kolmogorov's research interests in mathematical statistics (later he published a paper about it).
[7] It's not clear what he means exactly. Kolmogorov wrote a few articles on Lobachevsky at that time.

2) Family matters (bring Vera and Varya, send Varya to Nadya,[8] bring Anya from work and so on).

3) Maintain constant and steady work on big projects (analysis course, turbulence, spectra).

Only later — let mathematics join the variety of purely personal and general interests and hobbies that have flourished in the last two years.

Most important now:

1) Discipline in doing boring work.

2) Confident and consistent clearing [of tasks] to find possibilities for working calmly on big projects.

3) Fighting "temptations" (sweets, reading at the wrong time), including updating this notebook immoderately. (Agreement with Pusya to limit chatting!)

And where is love (Christian and non-Christian) I think a lot about and maybe talk too much about (for instance, to Oleg)? It seems that it is for the sake of love that I have to concentrate on disciplinary rules listed above!

Enough with reasoning! However, it's not prohibited to supplement records of labour deeds with short notes on moods and pleasures of life.

Saturday, 7 August 1943

It is a little misty, milk-sunny, warm day.

During the morning and after lunch I edited the first 15 pages of Gubler's work[9] (had to completely rewrite 7 pages). Satisfied with my work.

Pusya is listlessly writing on Lobachevsky,[10] editing Yura's[11] poetry, being upset about the Kazan troubles.

Marina was taken ill and Shurka drove her to a hospital in a horse cab accompanied by Spitsyn.[12]

A letter came from Sergey Musatov[13] telling us that he could come to Komarovka no sooner than in a month. He also mentioned that he wanted to write more to me about 'certain questions'.

FOUR WAYS TO COMPREHEND THE TRUTH

	Fantastic	Rational
Enthusiastic	1. Mystical hysteria	2. Ravishing piety
Skeptical	3. Shameless humour	4. Educated boredom

Sunday, 21 November 1943

Finally, on the way home from Dimetrusya[14] I resolved my some rather naive puzzle on what distinguishes characters from positive-definite functions (and why characters are generally positive-definite — yesterday I discovered that I had not understood that before!).

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[8] Vera, Varya, Nadya – Kolmogorov's aunts (his mother's sisters). He wanted to bring Vera and Nadya from Kazan where they had been evacuated to.

[9] Discussion on the paper *Solution for some cases of the shooting problem with intentional scattering* by I. A. Gubler. It was published in *Collection of papers on the theory of shooting* edited by Kolmogorov, Proceedings of the Steklov Mathematical Institute, Vol. 12, 1945, pp 67–94. — Editor, *Kolmogorov's Notebooks*.

[10] It might be one of the following: the book *Nikolai Ivanovich Lobachevsky. 1793–1943* by Alexandrov and Kolmogorov, a paper by Alexandrov on Lobachevsky geometry, or Alexandrov's notes for his public lecture about Lobachevsky that was held on the 6th of October 1943.

[11] Yura – Yuri Mikhailovich Smirnov.

[12] Spitsyn – a neighbour.

[13] Sergey Musatov – Anna Kolmogorova's first husband and Oleg's father.

[14] Dimetrusya – nickname of D. D. Romashov, a schoolmate of Andrey Kolmogorov, Anna Kolmogorova, and Sergey Musatov.

> Let $G = \{g\}$ be represented by operators U_g (unitary) in E'.
> Each $x \in E'$ corresponds to a positive-definite function
> $$\phi(g) = (U_g x, x).$$
> If $\lambda_1, \lambda_2, \ldots, \lambda_r$ are eigenvalues of U_g and
> $$x = (x_1, x_2, \ldots, x_r)$$
> in the corresponding coordinates, then
> $$\phi(g) = \sum \lambda_r x_r \bar{x}_r.$$
> Taking the average value of $\phi(g)$ over the sphere $S^r : \|x\| = 1$, we get
> $$M_{S^r}\phi(g) = \frac{1}{r}\chi(g) = \frac{1}{r}\sum \lambda_r.$$

What remains now is to find out what representation is induced by $\chi(g)$: for instance, is it always a representation in E^{r^2}?

My definition of a character seems to work for all locally bicompact[15] groups. However, Gelfand mentioned that, generally speaking, there was no analogue of characters as positive-definite functions.

Maybe that means $\chi(g) = M\phi(g)$ are discontinuous.

In all cases, the following is true (although the proof is not as straightforward as one would hope):

> If a representation in E^{kr} can be decomposed into k equal irreducible representations in E^r, then there is a vector generating all of E^{kr} if and only if $k \le r$.

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[15] Nowadays "bicompact" is just called "compact".