Mathematics K–12: Crisis in Education

Interview of Wu Hung-Hsi by Y K Leong

Wu Hung-Hsi

Introduction: Wu Hung-Hsi was the Singapore Mathematical Society Distinguished Visitor in 2010 and was in Singapore from August 30–September 4, 2010 to give talks to and interact with mathematicians, teachers and students. He gave an academic talk at the National Institute of Education and conducted two sessions of the Teacher's Workshop at the Teachers Network and the NUS High School of Mathematics and Science.

Professor Wu is currently Professor Emeritus at the University of California at Berkeley where he has been Professor of Mathematics since 1973. He is a well-known mathematician whose original research interests were in real and complex differential geometry. During the past two decades, he has devoted almost all his time, energy and mathematical expertise to improving K–12 mathematics education in the US. He started off as a part-time critic of the status quo of American school mathematics education in 1992, but he was soon alarmed by the quality of the available school textbooks and teachers' content knowledge, and especially by the way teachers were educated in the universities. His professional concerns soon crystallised into a personal mission to address the many issues that surround the mathematics education of teachers and educators. As a result, he is actively involved in numerous committees and panels concerned with mathematics curriculum, standards and the professional development of K–12 mathematics teachers. He has contributed his expertise and service to California's Mathematics Professional Development Institutes, the Mathematics Steering Committee of the US National Assessment of Educational Progress (NAEP) and the National Mathematics Advisory Panel.

On behalf of *Mathematical Medley*, Leong Yu Kiang interviewed Professor Wu on September 1, 2010 at the Department of Mathematics, National University of Singapore. The following is an edited and enhanced version of a frank and passionate interview in which Professor Wu gave the background of how he became an accidental, if only initially reluctant, advocate for changes in K–12 mathematics education in California. We also get to catch a glimpse of his views on modern mathematics and its influence on school mathematics education.

A Truth Embargoed

Mathematical Medley: When and how did you become deeply involved in the issues of mathematics education, especially in the teaching of mathematics in schools and the training of school teachers?

Wu Hung-Hsi: It was entirely an accident. I was happily doing mathematics, but in 1992 I had just finished writing a long paper — that paper took me two years to write — and I was taking a rest. At that point, my wife got a call from one of the local school board members — the board which oversees the local school district — who asked her, "Your husband is in the math department [of the University of California at Berkeley]. Does he know anyone who is willing to take a look at one of the new textbooks because this set of textbooks has been reviewed only by educators. Maybe we can get a mathematician to look at it." At that point, I was too tired and couldn't do any mathematics for a few weeks anyway, so I said, "Well, I'll volunteer myself for the review." I thought I would just go over there, take a brief look, say a few words, and then would be out of it. One or two days at most, and I would go right back to mathematics. So I promised to go over to the school district office to take a look at what kind of new textbooks they had.

When I was there, they said, "Well, we're going to pay you 2000 dollars to write two reports." I laughed because I didn't expect to write anything or get paid, but I was stuck because I had already promised to do it. They said, "It takes only one week to write one report; so it takes two weeks." At that point, I had no idea how to write such reports because I had never touched school mathematics education. But they explained to me, "Here are two new curricula. We'd like you to evaluate both of them. We will pay you 1000 dollars for each report." I thought I could still afford to spend two weeks, so I said OK. It turned out I could write one of the reports easily; it was very simple and it was easy to come to a conclusion. But the other one was a new set of textbooks that was still being field tested in America, meaning that it was being tried out in schools and they had not yet published the book. So they showed me a draft.

I took it, went home and started reading it. And I thought to myself, "I have never seen mathematics like that." To me, mathematics is very clear and very solid, with clear- cut theorems and clear-cut proofs. (You may call "theorems" and "proofs" by different names in the K–12 context, but the underlying fact remains the same.) Even in school mathematics, people should be able to say, "Here is the formula. You can use it to derive certain facts. I can give you the reason why it is true." But this textbook goes on and on. Everything is informal; it almost never gives a precise definition. It avoids using symbols as much as possible because it prefers verbal expressions over symbolic statements. Moreover, its exposition is often of the following variety: using a calculator, they get the answer to a real-world problem by some naive reasoning; but once the real-word problem is solved, they do not go back to give mathematical context to the reasoning behind the use of the calculator. In other words, this was not a textbook that brought mathematical closure to its mathematical discussions. It was not the kind of mathematics I had ever come across, and I asked myself, "What is it all about?" So I became very curious and also got very annoyed. I started to ask questions and interviewed one of the authors to get some information. After one month — and I didn't expect it to be that long because I wanted to finish the writing as fast as I could — I began to get the idea that something very serious (and not necessarily welcome) was underway in school mathematics education. Something new was upon us. I started asking my friends, even people I had never met before. I said, "What's happening?" But nobody seemed to know much about anything.

Then slowly I got to know that in 1989, the National Council of Teachers of Mathematics (NCTM) had published "Curriculum and Evaluation Standards for School Mathematics" (*NCTM Standards for short*). I think this document is known to Singapore. It has a new point of view about how to teach school students mathematics. So I bought a copy of the NCTM Standards and took a look. It was pretty strange; nevertheless, I got to know what they were trying to do. Then, after looking around for two months, I spent another month to write the report. Altogether, for 1000 dollars, I spent three months. I wrote what I considered to be an honest report. In fact, that report is still on my homepage. It's called "Review of the Interactive Mathematics Program (IMP)", http://math.berkeley.edu/~wu/IMP2.pdf. I gave IMP credit for doing some things right, but I also criticised it quite severely for what I thought they did wrong. I said that mathematics should not leave people in doubt as to what is true or what is not true. You either say it is true or false, or you can just say you don't know. You should not say something and leave it hanging, and then let students decide whether it is true or not true. That's not acceptable, and in any case that is not the kind of mathematics I know. I had no idea at that point that mathematics education could be completely mixed up with politics, to the point that the intellectual component of mathematics could be compromised by real-world considerations. I had been doing mathematics for 30 years at that point, but I was simply naive about school mathematics education. By now, my conclusion is that school mathematics education is 80 percent politics and only 20 percent intellect. Back then, I had made the mistake of confusing school mathematics education with the discipline of mathematics in academia.

My report is, in my opinion, a fair critique of IMP, and this opinion of mine was subsequently corroborated by other people on both sides of the so-called "Math Wars". But the authors of IMP didn't like it, and some school board members, who were politicians and for their political purposes wanted to support IMP, didn't like it either. The fact that I criticised IMP was not acceptable to them. So one day I got a call from the local school district, telling me that since they had paid for the report, what I wrote was their property and they were putting an embargo on it. They told me in no uncertain terms that they would prohibit it from being circulated. I was in shock because, in the world of research, when you write a paper you circulate it freely among your friends. I was in fact in the process of mailing my report to other mathematicians, asking them if they knew this was happening in school mathematics. I was stopped dead in my tracks.

I was not allowed to say a word about my report. It was a naked suppression of free speech, the most undemocratic act that could possibly be committed in a democracy. I was furious that anyone would dare suppress my freedom of speech. In addition, I was disturbed by the fact that an education document (the IMP textbook) could arouse so much passion that people would go to such lengths to stomp on my civil rights. I tried calling various lawyers and ended up talking to the legal counsel of Association of American University Professors (AAUP). The person was very nice to me even though I was not even a member. I told her everything. At the end, she said, "Fax me all the documents and I'll take a look." She called back in a day, or maybe two days, and said, "They [the school district] were bluffing because they have absolutely no right to embargo anything. It is your intellectual property. Do anything you want with it." I was relieved, but I also began to feel that something had gone wrong and I wanted to know why. That was the beginning of my involvement. I think, looking back, that if they had not suppressed my freedom of speech, I would have written that report, sent it out to my friends, and forgotten all about it in a few weeks. But they embargoed my article and got me mad, and I wanted to find out why. The more I found out, the more serious I got about school mathematics education.

What happened was that there was a group of people who got federal grants to write new books. Subsequently, I got to know firsthand a few of these books in addition to IMP, and my judgment was that they (like IMP) misrepresent mathematics in the sense that they fail to do, in a consistent manner, one or more of the following:

(1) give precise enunciations of definitions and mathematical conclusions (i.e., theorems),
(2) provide reasoning to support each mathematical assertion,
(3) delineate the place of each concept or skill in the mathematical hierarchy, and present mathematics in a way that respects this hierarchy,
(4) show students the mathematical purpose of each concept and skill.

A comment about the last may be in order. Mathematics is a goal-oriented discipline, and each concept and skill in the standard curriculum is there to serve a mathematical purpose; students ought to be told what that purpose is because it would give them the incentive to learn it. It enraged me to see a group of people going out of their way to, so to speak, corrupt mathematics. If all they did was to corrupt mathematics as a favourite pastime, it was their business and it would have been within their rights to do as they please; I would have nothing to do with it then. But they were writing textbooks to educate the next generation and, in that case, I could not stand by idly while they used such books to educate the young.

M: Was that in California?

W: The books were used nationwide (in the US). The more I got to know about the general situation in math education, the more agitated I got. And something else happened too. I started to ask teachers and talk to parents, and I got phone calls. Some parents said things like, "My kids are not learning. What can you do to help me?" It was then that I became fully aware of how desperate the situation really was, the fact that I could not even refer to any textbooks or reading materials for them to learn some genuine mathematics. I found out that textbooks and the associated mathematics education materials had deteriorated to the point that they were unreadable. I felt as if I was in a nightmare and I was walking by a river and someone was drowning and yelling for help, and all I could do was stand on the river bank and watch in helpless horror. I was a mathematician and wanted to help, but unless I did some kind of tutoring myself — which was not practical — I could not help students by telling them to take a class or read a book. Good school mathematics education was, for all practical purposes, unavailable. Therefore, I felt that I had to do something.

M: That's surprising. Was that in the 1990s?

W: That was in 1992, January 1992. I wrote my report in April. The New Math was about 1960 to 1970. The "Back to Basics" movement started around 1975 or so in reaction to the New Math. It basically reduced school mathematics to procedures without reasoning, and the NCTM Standards were, in turn, a reaction to "Back to Basics".

Teachers' Standards, Textbook Quality

M: You are writing some books yourself, isn't it?

W: There are several reasons for writing books. The first one is that as soon as I got involved in school mathematics education, I realised that there was a real crisis in teachers' content knowledge. At the beginning, people already told me that school textbooks were no good and students could not learn from them. Of course, as a mathematician, I had to make my own judgement in addition to listening to people. I took a critical look at the existing textbooks and sure enough, they were bad. But I also saw that if teachers knew enough mathematics, they could probably smooth over some of the rough spots in textbooks. But our teachers did not know enough mathematics, therefore they could not help students to make mathematical sense of the textbooks. So the first thing I wanted to do was to teach teachers mathematics.

I should tell you something funny. I thought it was simple to teach teachers: the minute I told them I was willing to teach them for free, they would all flock to me! At the time, I was so naive that I had no conception of an average teacher's workload, the fact that (American) teachers were (and are) all overworked. I told one of the local teachers, "Look, I'm willing to teach free of charge. You name the time; I'll come to teach you. If you can get me a group of teachers who want to learn, I'll teach them all the mathematics they need to know." In a very roundabout way, she tried to explain to me why my offer would fall on deaf ears. The teachers worked so hard during the school year that they could not afford to spend extra hours to learn new things. As to summer, although it was their vacation, some would need the time to unwind, while others would use it to earn the needed extra income. Even if I had proposed to teach them for only two weeks in the summer, they would not have given me two weeks. Thus regardless of the fact that I knew teachers' quality was the real issue, it took a long time before I could get directly involved in teachers' professional development. The opportunity was eventually offered to me in year 2000, and the minute I started to do that, I knew I had to write my own materials for teachers because the quality of the available textbooks was incredibly low.

Coincidentally, by 1999, I had become very involved with the real politics of mathematics education in California. For some reason, the state government trusted me at that point, and I was involved in several official state functions, including being one of the people in charge of the state's school textbook adoption. California is one of the few states that has a textbook adoption program, which means that all publishers have to submit their textbooks for approval by the state and the state has to decide whether those books are worthy of adoption or not. Strictly speaking, none of the books in 1999 was good enough, but of course we had to pick the best among the available ones because students needed textbooks. At that time, California had just adopted a new set of standards and we wanted to use that occasion to demonstrate how mathematics should be taught. So the state told the publishers about the raised expectations. As a result of my official responsibilities, many publishers' representatives came to see me and asked me what qualities California really wanted to see and "the right way" to teach this and that. It was then that the reality hit me: there was no book to which I could refer them and say, "Read it and do better." It became clear that some reasonable books about K–12 mathematics had to be written.

M: Wasn't there some kind of department of education?

W: It's not like that. In Singapore, the Ministry of Education decides everything. In the United States, the situation is very complicated. First of all, the United States has 50 states, but because there is no mention of the role of the federal government in school education in the US Constitution, school education becomes the local responsibility of each state. So the above-mentioned textbook adoption policy is applicable only in California. I should add that since California is such a big state, anything that California does carries weight and therefore the California adoption could have some repercussions in other states.

I'm writing a set of books for teachers. The first book is for elementary school teachers. [Note added September 6, 2012: it has since appeared as *Understanding Numbers in Elementary School Mathematics*, American Mathematical Society, 2011] Two other sets, a volume for middle school teachers and a three-volume set for high school teachers will follow. (In America, *middle school* is grade 6 to grade 8 and high school is grade 9 to grade 12.) Together they constitute a series of textbooks for K–12, in the sense that they develop mathematics in complete accordance with the school curriculum. But they are written for teachers, so there won't be much baby talk because their target audience is teachers, not students. I want to stay away from writing books for school students because, in such a book, the pedagogical and psychological components will be heavier. I don't think I want to deal with that. If I do that, I will get shot, for multiple reasons! The main thrust of this series is to give an exposition of school mathematics that is mathematically correct. This may sound simple to do, *but it is not*.

Publishing student textbooks in America is very, very complicated. They have to be politically correct, psychologically acceptable, agreeable to local school boards of education, and that sort of thing. Secondly, it is not enough to write the books for students. You must also write another set of books for teachers, telling teachers how to use the student-books. Then you must have solutions to the problems in the student-books. On top of all that, you must also prepare the so-called *classroom materials*, i.e., expensive graphics, worksheets, and all kinds of tools. It's a big industry. I don't think in my lifetime I will ever publish anything like that. I think I have a good enough understanding of school mathematics, and I want to tell people how it can be developed correctly in the *mathematical sense*. I want to convey this message, and I don't want people to tell me, "Is it politically correct? Is it psychologically acceptable?" I do mathematics, and that's enough.

In-service Professional Development, the Real Needs

M: You have probably answered part of the following question. What were the main obstacles that you faced when you first tried to communicate with elementary and middle school teachers on the most fundamental aspects of mathematics?

W: That is a very good question. I hope my answer will be useful to other mathematicians who want to contribute to school mathematics education.

When I first started to teach teachers, I made one fatal mistake. I treated them the same way I usually treat my undergraduate students. But they are not the same. When I taught an undergraduate course, I would tell students on the first day: "This is what I'm going to cover, this is my grading policy, and these are my expectations on homework and exams." It would be understood that I was setting a standard. Students who didn't like what I had to offer could take another course, but if they stayed on, then they knew they would either meet this standard or flunk the course. Well, this approach is not going to work very well with teachers.

By now, I've been teaching in-service teachers (teachers who are already teaching in schools) for more than ten years and I have taught them in summer institutes lasting anywhere from one to three weeks. It took me a while to learn the difference between teaching in-service teachers and teaching undergraduates. *Assuming that the professional development is focused on mathematics*, then I can summarise the differences as follows:

(1) Undergraduates come straight from schools and are still stuck on the *learning mode*; they know they are supposed to learn. However, because in-service teachers are used to *teaching students* rather than being students themselves, their ability to listen and take in information has been somewhat compromised. This difference becomes pronounced when it comes to learning the subtleties of sustained logical arguments. When one teaches teachers, one should be sensitive to this difference.

(2) We teach undergraduates mostly mathematics that is new to them, and their mis-education in K–12 mathematics can sometimes be masked by the novelty of the subject matter. However, teaching the mathematics of K–8 to elementary and middle school teachers forces them to confront their mis-education and there is no escape: they must first unlearn what they were taught in school before they can pick up the new knowledge. Unlearning something can be very difficult.

(3) The human factor: In-service teachers have been teaching for years and are used to passing judgement on their students, but when it is their turn to be judged, there's a delicate issue with ego that simply cannot be ignored. The same ego problem with undergraduates is by comparison quite trivial.

(4) If we think of university courses as obstacle courses for undergraduates, then we accept without a moment of thought the fact that some would be left behind after each obstacle. But in teaching in-service teachers, one has to banish the thought of "leaving anyone behind" because any teacher "left behind" in this sense will end up damaging thousands of students in his or her career. One must try everything possible to bring every teacher along, no matter how difficult that may be.

But even these differences do not tell the whole story. One should ask instead why we are talking about teaching teachers *the knowledge they should have gotten as part of their education* in the first place. The reason is simple: the whole education establishment has failed them every step of the way. In the universities in America, we don't teach teachers any mathematics they really need for their job, which is to teach school mathematics. That is the absolute truth. We have university mathematics, i.e., calculus, discrete mathematics, abstract algebra, analysis, differential geometry, numerical analysis, whatever. You name it, and I can tell you why it is — in principle — good for our future teachers to learn. But knowing all that will not directly help our teachers teach school mathematics.

In fact, I've written an article to that effect. [Note added 6 September 2012: The Mis-Education of Mathematics Teachers *Notices Amer. Math. Soc.* 58 (2011) 372–384, http://math.berkeley.edu/~wu/NoticesAMS2011.pdf.] We have not given teachers the knowledge they really need to teach school students. The material in school mathematics (K–12) is not covered at all in regular university mathematics courses.

M: But they are supposed to have learnt that in school.

W: It's a vicious circle. As I told you before, school mathematics education has been very bad in America for a long time. When students get out of school, they don't know mathematics, only what is contained in school textbooks, which is unfortunately a flawed version of mathematics. When they come to the university, they expect to get help. But no help is forthcoming because our universities make believe that if you graduate from K–12, you already know school mathematics and there is no need to talk about it. Thus by the time the undergraduates go back to schools as teachers, they know exactly as much about school mathematics as when they graduated from school. So the flawed version of mathematics gets recycled from generation to generation.

I have not done any research into other countries' education systems to know whether the phenomenon I have just described is strictly an American one. I suspect it is not. Something this important — the future of our children — clearly deserves a thorough study.

Instead of school mathematics, American universities prefer to talk about other things: for high school teachers, it is advanced topics like differential equations or groups and rings, and for elementary teachers, it is pedagogical strategies. However, in view of the deficient mathematical knowledge that students bring to the university, a more realistic approach to their education would be to hold their hands and tell them, "Be careful. If you're going to be an elementary school teacher and want to teach students how to multiply, can you explain what the multiplication algorithm is all about?" Now, think about it: which university mathematics course teaches its students, in a mathematically responsible way, why the multiplication algorithm is true? For example, if it is an algorithm (i.e., a finite machine procedure), what exactly does it say, and what is its main mathematical thrust? As for the long division algorithm, I don't even know if any book tries to explain, correctly, what the algorithm is supposed to accomplish and why it is valid. Another example: what does it mean to solve an equation? This is a much harder question than most people realise. Right now, all across the country in America, almost all teachers and textbooks teach incorrectly how to solve an algebraic equation.

If we want someone to be a good teacher, we have to carefully teach her the material she is going to teach. However, we seem never to have taken this simple truth to heart. We blame the teachers for not knowing mathematics, but the fact that most don't is almost completely the result of our own negligence. *We do not teach our teachers the knowledge they need for their job*. I came to the realisation of this fact early on in my involvement with education, so I was determined to teach mathematics to teachers above all else.

Now in-service professional development is big business in America. Lots of these professional developers make lots of money by going around school districts and telling teachers, "You pay me for a day (or two days) and I will guarantee you results." You know what most of them do? Their main objective seems to be to make teachers feel good. They pat teachers on the back and tell them, "Well, math is fun. We will discuss tricks to solve some special problems, and we will present you with some classroom activities. You will bring them back to your classrooms, and the kids will love you." There are also organisations that pay teachers to go through this kind of professional developments. Of course, I want teachers to feel good too and I want them to believe that mathematics is fun, but none of that means anything if they do not know the most basic things about what they have to teach. These professional developers do not address the bread-and- butter issues of teaching mathematics: What is a fraction? Why invert and multiply when you divide fractions?

It may not be a particularly good analogy that I am going to give, but I will give it anyway. Suppose a group of people are dying of starvation and you come to their aid. What do you think should be the primary obligation?

Give them the most basic items like vegetables, rice and some meat, of course. But some people don't see it that way. They come to this group of people and tell them, "I'll teach you how to make a soufflé." This is the analogue of what most professional developers are doing to teachers. And it's so much fun to make a soufflé, don't you think? But that's not what starving people need. Starving people need the most basic ingredients in order to regain their strength and carry on with their lives. Our teachers are starving for knowledge in order to teach well but, for some reason, people don't want to take this need seriously. My proposal is that, above all else, we have to provide teachers with this basic mathematical knowledge. This is my goal, to deliver this knowledge. You may find it a bit strange, but it is a fact that many educators' reaction to this proposal is that "teaching is complex, and knowing math is not everything." Life is complex too, and being free from the threat of starvation doesn't solve the profound problems of life either. All the same, if you have to worry about starvation every minute of the day, it is hard to imagine you can solve any real problems in life. So until we can provide teachers with adequate content knowledge, math education isn't going to go anywhere. At the moment, there doesn't seem to be any systematic program in America to provide teachers with this basic content knowledge, yet the education establishment claims to be doing its best to solve the math education crisis. It is strange, no?

"New Math" and Mathematics Reforms

M: In the 1960s, the "New Math" was incorporated as an important part of the school mathematics curriculum in the US and perhaps around the world. Was this due largely to the enthusiasm and optimism of mathematicians at that time in the power of abstraction and axiomatisation?

W: That certainly played a part. You cannot deny that. However, the seeds of the New Math were planted in the early fifties by an educator-teacher, Max Beberman. Nowadays, we tend to identify the New Math with School Mathematics Study Group (SMSG), headed by Ed Begle (a mathematician), because SMSG was the unit officially entrusted in the early 1960s with the national reform of the school mathematics curriculum. SMSG was backed by an unprecedented amount of National Science Foundation (NSF) funding, and its experimental textbooks were written by teams of mathematicians and teachers. Certainly, the enthusiasm and optimism of mathematicians about Bourbaki's work affected the New Math people — excessively in some cases — in their attempt to update an antiquated school curriculum and textbooks. It did not help that the people in charge did not seem to have a firm grasp of how much formalism and abstraction would be appropriate in school mathematics or, for that matter, could be tolerated by school students. One recalls in this connection the often pedantic distinction between a number and a numeral, or the insistence on the use of the language of set theory. More importantly, although the New Math people anticipated the need to educate all the teachers (especially those in K–8) about the new mathematics they were to teach, they were no match for the onslaught of mathematically illiterate textbooks put out by publishers. In the end, vastly more teachers were educated by these illiterate books than by the SMSG program. The lethal combination of ill-informed teachers and illiterate books was what did in the New Math.

M: Was the "Back to Basics" movement that followed in the 1970s a reaction against the "New Math" and did that movement achieve anything?

W: Well, I'm not an expert on the "Back to Basics" movement. The little bit I know is that it did not achieve very much other than rectifying some blatant errors in the New Math. For example, Back to Basics at least restored the standard formulas in the school curriculum so that students could do basic computations fluently. On the other hand, I think that the Back to Basics movement over-emphasised the procedure aspect of mathematics at the expense of reasoning and coherence. That was a fatal error.

M: It must be a highly non-trivial task for a mathematician who is not formally trained in pedagogy to train mathematics teachers. How much of it is about effective teaching and how much is it about subject content?

W: Mostly about subject content but ultimately both. Let me explain. In teaching teachers, the choice of the content is not standard mathematics, but mathematics that has been engineered to be usable in a K–12 setting. (See H. Wu, How mathematicians can contribute to K–12 mathematics education, *Proceedings of International Congress of Mathematicians*. Madrid 2006, Volume III, European Mathematical Society, Zürich, 2006, 1676–1688, http://math.berkeley.edu/~wu/ICMtalk.pdf.)

This process of engineering includes considerations of how to optimise the presentation so as to make the content most accessible to school students of a particular grade level. Such considerations are necessarily pedagogical in nature. People usually have the impression that because mathematicians only do mathematics, they don't know pedagogy. That's often not the case. Many of my colleagues, at Berkeley and elsewhere, are extremely good teachers and they certainly know enough about pedagogy for their needs. Mathematicians should not pretend to be experts in pedagogy, but that doesn't mean they have nothing to offer about pedagogy.

M: Are the latest reforms in mathematics education characterised by a call to return to concepts and proofs, perhaps a "Back to Rigor" type of movement?

W: I think a crucial mistake made by the New Math leaders (most, but not all, were mathematicians from universities) was that they were not sufficiently knowledgeable about schools. By contrast, the mistake that was made by educators in the latest reform was that they did not make an effort to learn the necessary mathematics. If you do mathematics education, you have to do two things: You have to know both mathematics and schools. It's not enough to know just mathematics, and it's also not enough to just know schools. It's most unfortunate that, in America, there has been very little dialogue between mathematicians and educators. This non-communication stands in the way of any real progress in mathematics education. No kind of education would be any good if the educators of that particular discipline (e.g., mathematics) become divorced from the professionals in that discipline (e.g., mathematicians). What I have been trying to do in the last ten years is to keep reminding them that they have to come back together. Each has to learn from the other and they have to collaborate. I myself keep a constant dialogue with teachers.

Mathematicians, Computers and Schools

M: Are there many university mathematicians who go to the schools and talk to students and teachers?

W: There are, but not many. Among research mathematicians in America, how many of them are interested in school mathematics education beyond the local level? Very few. For this reason, few research mathematicians would go to the schools and give talks — no reason to do so — unless they are interested in improving school mathematics education. Of course, it can happen once in a lifetime that you get to talk to school students because someone comes to you and asks, "Would you like to talk to a bunch of school students?" Otherwise, the political aspect of school math education tends to discourage mathematicians' participation. As a fellow geometer once told me, school math education is a bottomless pit.

M: I think Serge Lang used to go to the schools to give talks.

W: He had time on his hands. He wrote several books, including one on plane geometry and one on Basic Mathematics. In my opinion, they are good books for school students who are interested in learning something. Some teachers read them but didn't like them because they are too brief. I think that is a fair assessment; they are quite hard. Even for teachers they are hard because a reader has to work quite a bit to fill in the gaps in the exposition. Serge liked to improvise and he pretty much did that when he gave talks to students. I think he did it more or less for fun. He would have been quite upset to hear me say this, but I don't think he approached school education as a professional, in the sense that if he was serious about it he would have tried to first change the school culture. I don't think he did that. He liked to argue and criticise, but I think criticism is not that helpful to teachers at this juncture.

M: School teachers are often not comfortable with proofs of theorems. Would it help if they are taught a course in logic that uses examples in basic arithmetic and geometry?

W: I don't think so. Let me give you an analogy. Suppose someone doesn't write English well. What do you think should be done: give him good reading material and make him read a lot and then point out to him why the writing is good, or simply make him read a book on grammar? It's basically the same question. Don't let school teachers go to logic directly, but let them learn logic in a context of substantive mathematics. Then they will have a much better chance of learning it. Just give the teachers a well written math book on the right level, one that has substance and explains mathematics clearly and logically. Let them learn some interesting mathematics from the book first, and then focus on a few good theorems and analyse a bit about what each theorem says and how the proof serves to reveal why the theorem is true. Use these proofs to show them that the purpose of a proof is to go from Point A (the hypothesis) to Point B (the conclusion) by the use of logic. All of mathematics is nothing but going from Point A to Point B. If teachers can learn this fact from studying interesting theorems, then they get to see logic in action. This would give them a better chance of learning what a proof is. I don't believe studying logic per se would be all that is helpful for most people.

M: Your own mathematical research is in real and complex geometry. Did S S Chern [(1911–2004)] have any influence on your choice of specialisation?

W: It may seem surprising, but I did not decide to do differential geometry because of Professor Chern. Rather, I made up my mind when I was an undergraduate that geometry was the only kind of mathematics I could do. I met him later, of course, and we were colleagues for about 30 years. Professor Chern was 30 years older than I, and I have written two articles about him since his death. He definitely had an influence on me in terms of my overall mathematical outlook. By the way, it was not only Professor Chern, but S T Yau also had the same influence on me. My thesis advisor was [Warren Arthur] Ambrose, who learned a lot of differential geometry from Professor Chern's writing. (But Ambrose always claimed that he couldn't understand Chern's writing and had to reconstruct all of Chern's arguments.) Ambrose was already an established mathematician when he switched fields from functional analysis to differential geometry. I guess I was a figurative "grand-student" of Professor Chern. I don't believe I ever learn much differential geometry directly from him, but what I did learn from him is far more important: mathematical taste and judgement. As to S T Yau, he was Professor Chern's student, of course, and he also took a course from me as a student. Both taught me that when you do mathematics, you look for what is important and ignore everything else. With this attitude, you focus on the central issue and let the lesser issues pass you by. That's pretty much how I approach mathematics education. I'm using what I have learned from these two people [Chern and Yau].

M: How much have the advances in computer visualisation contributed to the teaching and understanding of geometry at the elementary and middle school level?

W: My opinion on this issue is not professional, in the sense that I have not done any research [on this] and I'm not a psychologist so that I cannot say anything on the subject with authority. That said, I do not believe that visualisation by computer would initially help students learn geometry. I think, at the beginning, the only way to learn geometry is to draw a lot of pictures by hand. My opinion is shared by quite a few others — learning goes from the finger tips to the brain — and it seems to me there's no other way. You have to draw pictures or maybe build models because, at the beginning, the kind of geometry you learn is two-dimensional or three- dimensional. Of course, eventually when you go to higher dimensions, you may need the computer to make models or help to think about things; for example, the three- dimensional projections of the six-dimensional Calabi– Yau manifolds. But for middle and elementary school, I think it's better to do it the hard way by hand. I think there's no short cut for that.

M: But certainly, there's a lot of software for geometry, for example, software that architects use for three-dimensions.

W: What architects need is a very good visualisation of what a building will look like before it is built, so the software is important for an accurate rendition of the image. But for geometry, I think people are mistaken about the need for very accurate drawings. In general, all they need are rough pictures that are essentially correct. For example, I know something about two-dimensional Euclidean geometry, but I almost never use a ruler or compass when I try to prove a theorem. I draw circles or lines by hand to get a rough picture, and that's usually good enough. I don't think accuracy is so critical.

M: What is your latest project in mathematics or mathematics education?

W: I've mentioned that I'm writing a set of textbooks for teachers at all levels. If I have time I would like to also write a book on the history of mathematics for teachers. I feel that teachers should know something about the history of mathematics, but most books on the subject are more concerned with history — what I consider to be boring historical details — rather than with mathematical ideas. Teachers should know about the evolution of the great mathematical ideas such as the emergence of algebra in the Middle Ages, the impact of the Parallel Postulate, the evolution of the base-ten numeral systems, the concept of limit through the ages, etc.

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