M S Narasimhan, The Man and the Mathematician — A Personal Perspective

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M S Narasimhan is one of the most versatile Indian mathematicians of the post-independence era. He has an impressive collection of honours: The Bhatnagar award, Chevalier d'ordre national du merite of France, Fellowship of the Royal Society, London, Third world Academy prize, Padma Bhushan, King Faizal prize, ... It is a great achievement on the part of the individual who, in his student days had to commute to his High school, miles away, on a bullock cart. He comes from a landed family in a rural part of the district of North Arcot in Tamil Nadu. After schooling, Narasimhan moved to Madras and joined the reputed Loyola college. The head of the Department of Mathematics at that time was one Fr Racine, a Jesuit from France, who had obtained his doctorate under the superivision of the illustrious geometer, Elie Cartan. There was also another teacher, Professor Krishnamurthy, in the department and Narasimhan's interest in advanced mathematics was kindled by the courses offered by them. On finishing his undergraduate degree, he joined the Tata Institute of Fundamental Research (TIFR) as a graduate student in 1953.

I shall give here a personal perspective of Narasimhan's work with particular reference to my interaction with him.

I first met M S Narasimhan over fifty-five years ago. I had heard of him a couple of years earlier when a college senior, Sridharan, who had also joined TIFR, told me of Narasimhan's interesting work on elliptic operators. So when I, in turn, applied to TIFR for a position as a graduate student, I made it a point to meet Narasimhan after my interview and expressed my anxiety about the outcome. He simply shrugged his shoulders and said, "you should take these things easy!"

Soon after I joined TIFR as a graduate student, Narasimhan (as well as his colleague at college, Seshadri who had also joined TIFR) was sent to Paris for his PhD work. Laurent Schwartz had visited TIFR the previous year (and was incidentally also a member of the interview committee I have mentioned above) and given lectures on complex manifolds. He was impressed by the quality of the students there, and had taken pains to get the French establishment to award scholarships to TIFR students. Narasimhan made the best use of this visit, mathematically, culturally and socially. Paris was then the centre of the mathematical universe. Narasimhan and Seshadri had the great opportunity to learn and discuss mathematics from such greats as Serre, Chevalley, Cartan, Schwartz, Leray and many others.

Naturally, Narasimhan came under the influence of Schwartz, although I understand that Schwartz used to live far away from Paris near Orleans, and came in only twice a week. Students would line up to meet and have discussions with him on those days. Early on, during his stay, Narasimhan had the misfortune to contract pleurisy and had to be hospitalised. In all these years, I have never heard Narasimhan talk bitterly about this circumstance. On the contrary, he used to tell me that his stay in the hospital for a few months enabled him to interact with people of different social strata, which would have been impossible had he continued to stay at the Maison de l'Inde! He got an understanding of the problems and expectations of different classes, and this strengthened his leftist sympathies, already kindled by Schwartz (who was a staunch leftist — indeed a Trotskyite). He also felt that this interaction with the "real France" enabled him to improve his French, particularly the spoken language.

From the mathematical angle, he used the opportunity of his stay in the hospital to widen his knowledge of mathematics, unfettered by any formal requirement. In particular, he internalised the ideas of Kodaira and Spencer, which the lectures of Schwarz at TIFR had prepared him for. He later collaborated with the Japanese mathematician, Kotake, another student of Schwartz,

proving a beautiful analyticity theorem for solutions of elliptic operators that satisfy certain natural, Cauchy-type inequalities. He also completed his thesis there and submitted to the Bombay University.

On his return to Bombay, he found a graduate student in me, still struggling to find a good problem to work on, although I had published a couple of papers on homological algebra, one of them with Sridharan. Narasimhan warmed up to me when he saw that I was familiar with the work of Kodaira and Spencer, which I had come to know of from Hirzebruch's book, "Neue Topogische Methoden in der Algebraichen Geometrie". When I told him that I was not conversant with the analytical side of things, and mentioned the semi-continuity theorem, he gave me a crystal clear survey of the theory of elliptic operators in just over an hour. Later, we ran a seminar on the moduli of curves (with the help of another colleague of ours, R R Simha), broadly from the analytic point of view.

One evening, as I was waiting for my turn at the tennis court, Narasimhan came up to me and said, "You have learnt a lot of good mathematics. It is time to put it to use and you ought to work on a nice problem". Like any graduate student, I found it difficult to identify a reasonable problem to think about, which is interesting on the one hand and "abordable" (as Narasimhan used to say) on the other. But I said, "Well. Koszul visited us here and I learnt some differential geometry. I wondered if, just as there is a universal vector bundle on the Grassmannian, which of course admits a homogeneous connection, the connection itself is universal in some sense". He got very excited and we started working on it from that night on. He asked me various questions, like, "Did you check it for line bundles? Did you check it for the trivial bundle?" and so on. The methodology of research, where one looks for analogues, particular cases, etc. came naturally to him and it was a revelation to me. We worked on it for hardly ten days and we had our first joint work, "The existence of universal connections". It was sent to the distinguished differential geometer, Chern, who responded immediately with great enthusiasm. We had proved it for compact Lie groups, and followed it up with another paper where we proved it for all Lie groups.

Soon thereafter, Narasimhan collaborated with Seshadri and proved the famous result which mediated between the transcendal construct, namely the fundamental group, and the purely algebraic notion of stable bundles. This path-breaking theorem, with all generalisations and analogues, has been at the centre of various aspects of algebraic geometry and number theory, for over half a century.

A few years later, Narasimhan and Seshadri were invited to the UK under a British Council programme and it so happened that I was also to take up a post doctoral position at Oxford. We set off on the same flight. It was an eventful flight since the cockpit screen broke thanks to the turbulent weather condition and the flight returned to Beirut soon after taking off. We stayed in a hotel in London provided by Air India and they went on to Liverpool and I to Oxford. Soon after we returned, Narasimhan and I started working on the moduli of semi-stable vector bundles on a curve, which had just been constructed by Seshadri. We first determined the singular locus of these varieties. It is natural to expect that this is actually the non-stable locus in the moduli. This is what we proved, but the surprise was that there was an exception. We saw that if the curve had genus 2, and one considered the moduli of vector bundles of rank 2 and degree 0, the moduli space was actually smooth. This made us wonder what was this smooth variety and we checked that when the determinant was also trivial, it was just the three-dimensional projective space. The argument was a bit involved and we were somewhat dissatisfied. We began to ponder over the nature of this projective space. A few months of hard work led us to the (now satisfactory) proof that there was a canonical isomorphism of the moduli space with the complete linear system of twice the principal polarisation on the Jacobian of the curve. The result was thus submitted for publication a year after it was first proved. I relate this in some detail just to say that firstly, in the time and place where we worked, one did not have to rush to publish, thanks to the enlightened academic viewpoint of those at the helm of mathematics at TIFR, namely, K Chandrasekharan and K G Ramanathan, and secondly, that the ideas going into the "eventual satisfactory proof" proved very essential for the progress of the subject, as opposed to the result which was just "interesting".

About this time, Narasimhan also collaborated

with Simha to prove an interesting result regarding the existence of the moduli of complex structures of general type on a given real analytic manifold. Here, his expertise on the analytic side came to the fore.

Soon after the above joint work with Narasimhan, I went on a post-doctoral visit to Harvard, while Narasimhan visited the Institute for Advanced Study, Princeton, at about the same time. During one of our telephone conversations, he said that he had proved an (Euler Poincaré) result, which implied that discrete series representations can be realised as natural actions on homogeneous bundles on Hermitian symmetric spaces. This was done in collaboration with the Japanese mathematician, Okamoto. This was a total surprise to me since I knew that Narasimhan was no expert on representations of real Lie groups. Narasimhan used to say that one should work "off the top", and he displayed this strength of his again and again throughout his career. He would quickly get a working idea of a problem and could think creatively without worrying about the foundational aspect at first. One filled that knowledge later! Okamoto remarked that he learnt above all, how to engage in research from Narasimhan!

Narasimhan and I worked together intensely the next few years on various aspects of the moduli space of bundles. When Bertram Kostant visited us, he commented that he had never seen two mathematicians collaborate all the time as we did! Indeed, even many of our students had Narasimhan and myself as joint supervisors!

I was away at the Institute for Advanced Study, Princeton, and later at University of California, Los Angeles, for a couple of years in the late seventies. Towards the end of my stay there, I got the incredible news that Narasimhan had collaborated with a bright physics student at TIFR and got a very interesting result on an obstruction to Gauge-fixing. This student, T R Ramadas, had obtained an engineering degree from the Indian Institute of Technology at Kanpur, and had joined TIFR as a graduate student in Physics! He began discussing mathematics with Narasimhan and they soon found that the problem in physics they were interested in could be formulated in terms of the Gauge group action on the space of connections, and this led to the above result. This is another example of the ability of Narasimhan to quickly get to the essentials of a problem rather than get bogged down by baggage irrelevant to the problem on hand.

When I returned from my sabbatical, I became the Dean of the School of Mathematics at TIFR. There was then a demand in India that the time had come for the creation of a body for the development of higher mathematics, analogous to the Council of Scientific and Industrial research. The Chairman of the Department of Atomic Energy discussed this with me in my capacity as Dean, and decided that since mathematical research was flourishing in India under the Department of Atomic Energy, the demand could be met without disturbing the status quo, by the creation of a National Board of Higher Mathematics. Narasimhan was the first chairman and I, the secretary. I thus had first hand knowledge of his administrative acumen as well. This ability was even more in evidence when he later became the head of the mathematics activity in ICTP. His influence, both direct and administrative, lifted the mathematics wing there to a dynamic one.

Towards the end of my second sabbatical, as a Visiting Professor at Harvard, I was invited to a meeting in Maine. Narasimhan was also a speaker at the conference. We had just learnt about Hitchin's work and also about a result of Beauville which had used ideas similar to ours. In hardly a week of intense work there, we managed to put the two things together. A missing piece had just been proved by Laumon. Beauville had also noticed that the three ideas led to a nice result on the Moduli variety, and led to the first complete result on the Verlinde formula. Rather than publish them independently, the three of us decided to write it jointly.

Since his retirement from ICTP, Narasimhan has settled down in Bangalore, but he continues to be academically active. Narasimhan is an extrovert and freely exchanges ideas with others. This explains in part how he had a large number of collaborators such as W Decker, J-M Drezet, G Elencwajg, G Harder, A Hirschowitz, Y Holla, Shrawan Kumar, H Lange, J Y Li, M V Nori, T R Ramadas, A Ramanathan, F-O Schreyer, G Trautmann and others. In addition, there are many others who benefited just by talking to him.

Apart from mathematics, which is a passion with him, Narasimhan has also been interested

in other domains, such as classical Indian music, contemporary art (particularly painting), literature (including present day writings in Tamil), etc. Several of his students, including Nitsure, Patodi, Raghunathan, Ramadas, … have since distinguished themselves as first rate mathematicians.

I have had the privilege of working with him for several decades and have been in close contact with him mathematically and on many matters, administrative and even personal. I cherish my friendship with him and I am happy to use this opportunity to share some of my perspective with others.

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