Interview with M S Narasimhan

Sujatha Ramdorai

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Professor Mudumbai Seshachalu Narasimhan is a highly accomplished Indian mathematician whose seminal work in Algebraic Geometry is recognised worldwide and has made inroads into different areas within mathematics and theoretical physics. He was with the School of Mathematics, Tata Institute of Fundamental Research (TIFR) for a large part of his career and then was the Head of the Mathematics group at the International Centre for Theoretical Physics (ICTP), Trieste from 1992–1999. He currently lives in Bangalore, India. His work has fetched him numerous accolades and prizes, in particular the Shanti Swarup Bhatnagar Prize (1975), the Third World Academy Award for Mathematics (1987), Padma Bhushan (1990), Fellow of the Royal Society and the King Faisal International Prize for Science (2006; jointly with Simon Donaldson, Imperial College).

His eightieth birthday was marked by mathematical conferences in Spain (http://www.icmat.es/congresos/isc2012/) in September 2012, and in Bangalore (http://math.iisc.ernet.in/~imi/ICAG.php) in December 2012. Professor Narasimhan graciously consented to an e-interview with Sujatha Ramdorai in November which was followed-up with a subsequent tête-à-tête in Bangalore.

Sujatha Ramdorai: At the outset, my warmest greetings on your 80th birthday year. You have had a long and illustrious career as a mathematician. Looking back, what would you say were your best moments that you cherish?

Mudumbai Seshachalu Narasimhan: The best moments, I think, were the time I spent, as a student, with Fr Racine, K Chandrasekharan and L Schwartz, which shaped my approach to mathematics and my mathematical career.

SR: Can you tell us a little more about your childhood, the environment at home, your schooling ...

MSN: I come from a small village from Tamil Nadu, (from the now nonexistent North Arcot district) and the nearest secondary school was 5 miles away. I come from a family of agriculturists who were once fairly well off and due to droughts and my father passing away when I was about 12 years old (I was the eldest son), the family was facing reduced circumstances. I was good in my studies, especially in mathematics. I was fascinated by Euclid and the thrill it gave me to solve "riders", thinking for oneself. Even in school I wished to "do research", though I am sure I did not know what it meant. I was encouraged and supported by my family when I wanted to study mathematics and was not under pressure to pursue any other career. When I used to draw mathematical diagrams on the walls of the house, I was presented with a blackboard.

SR: And your college years?

MSN: I studied in Loyola College, Madras and it was a great fortune that Fr Racine was teaching in that college. He was in touch with several outstanding French mathematicians. He was attempting to introduce several modern fields of mathematics to Indian students and mathematicians. He was one of the first in India to introduce Modern Algebra at the undergraduate level. Association with him at the formative stage was crucial for my future mathematical development. He instilled in me a taste for good mathematics. It is he who suggested to me to go to the Tata Institute for pursuing research.

SR: Fr Racine was one of those legendary names one heard in TIFR often. The generation of Mathematicians in India who were directly his pupils clearly have warm memories. Could you tell us more about him? Did you stay in touch with him later during your mathematical career? What were his feelings about seeing people under his tutelage do so well in Indian (and international) math?

MSN: Father Racine was a member of the Society of Jesus and he spent the later part of his life in India teaching mathematics, first in St Joseph's college, Tiruchi and then in Loyola college, Madras. He obtained his doctorate in Paris, working with Elie Cartan. He was in touch with outstanding mathematicians in France, like Leray, H Cartan and Weil and was following mathematical developments in France and tried communicating them to Indian mathematicians. For instance, he lectured on the theory of sheaves in Madras soon after the work of Leray was published and his reviews of this work appeared in Zentralblatt MATH. He had a remarkable capacity for identifying students with talent and aptitude for mathematics and mentoring them. He guided his students in acquiring a broad-based training in mathematics and somehow enabled them to acquire the faculty to discern what is deep in mathematics.

He took particular interest in advising his students in the pursuit of a career in mathematics and also in following their progress. I used to keep in close touch with him and write to him regularly about what I was doing in mathematics. (It gave him and me special pleasure that we could correspond in French during my stay in France). In the later years I got to know him well and he liked to talk about mathematicians and scientists and he had a nice sense of humour. I visited him in a hospital in Bangalore during the last days of his life. Undoubtedly Fr Racine played a major role in the development of mathematics in India. The list of outstanding Indian mathematicians he mentored is impressive. Among his former students were: Minakshisundaram, K G Ramanathan, Seshadri, Raghavan Narasimhan, C P Ramanujam, Ananda Swarup and myself. He was happy and pleased with his role in starting the research career of so many first rate mathematicians and also, I am sure, with their sense of gratitude towards him. I should also mention that he was honoured by the French Government by a Legion d'honneur.

SR: You were one of the early members of TIFR (Tata Institute of Fundamental Research). Can you reminisce a little about the mathematical scene in the country at that period?

MSN: At that time (mid 1950s) there was a small number of good mathematicians in India, working in isolation. However, there was no expertise in India in many important fields of modern mathematics. ("Modern" algebra and topology were taught only in one or two universities!) There was no organised support for research nor a proper mechanism for training and channelling the talents of young Indians into creative research in Mathematics. The situation changed after Independence and the Indian Government made available substantial financial resources for the organisation and development of scientific research. The scheme initiated by K Chandrasekharan in TIFR to develop a School of Mathematics at the highest international level, was the turning point for mathematicians and I was one of the early beneficiaries of this development.

SR: The Narasimhan–Seshadri theorem was one of the results that put TIFR on the international Mathematical map. When did you actually start working on this problem? Can you tell us about how Weil's work eventually led to your work with Seshadri?

MSN: Already in our student days Seshadri and I were familiar with the paper "Generalisation des fonctions abeliennes" by A Weil. We became aware through K G Ramanathan of this paper which was pointed out to him C L Siegel. By 1960, the theory of vector bundles in topology was well developed and algebraic vector bundles were being studied by Weil, Serre, Grothendieck and Atiyah. It seemed to be an opportune moment to undertake an intensive study of vector bundles on projective varieties. My impression is that we had the problem of vector bundles on curves in our mind from our student days when we became aware of Weil's paper, but started thinking about it seriously in early 1960s when we familiarised ourselves with the theory of deformations of complex structures. From the present day point of view, Weil envisages in this paper a study of holomorphic vector bundles on a compact Riemann surface and attempts to construct their moduli spaces, generalising the construction of the Jacobian (the word "vector bundle" is not found in the paper and Weil works with "matrix divisors" or "adeles" as we will say today). Weil mentions that holomorphic vector bundles arising from unitary representations of the fundamental group of the Riemann surface should play an important role. Seshadri and I first showed that bundles arising from (classes of) irreducible representations of the fundamental group in a fixed unitary group form a complex manifold, using the then emerging theory of Kodaira and Spencer on deformations of complex structures. We realised that the crucial problem was to give an algebraic characterisation of holomorphic bundles arising from unitary representations. We also felt the "Method of continuity" of Klein and Poincaré could help in proving such a result, once the algebraic condition was available. In the Stockholm ICM talk (1962), David Mumford, motivated by Geometric invariant theory, introduced the notion of a stable vector bundle and this turned out to be the sought-after algebraic condition. Seshadri and I proved, in 1964, that a holomorphic vector bundle on a compact Riemann surface arises from an irreducible unitary representation if and only if it is stable and of degree zero. We also proved a corresponding theorem for vector bundles of arbitrary degree by considering unitary representations of suitably defined Fuchsian groups. The proof used a combination of techniques from algebraic geometry, complex analysis, topology and partial differential equations.

SR: Obviously those early years of TIFR, especially the contacts with the French School shaped the mathematical landscape in India then and into a long future. You were amongst the early mathematical pilgrims, and spent time as a student in France under Laurent Schwartz. Tell us about that and also the mathematical scene in Paris in those years.

MSN: I was in Paris during 1957 to 1960. During this period Algebraic geometry was being revolutionised by Grothendieck and others; the Cartan and Chevalley seminars were also taking place. There was also much activity in Paris on Partial Differential Equations by the school of Schwartz, streamlining and advancing the subject by a systematic use of the theory of distributions. I was interested at that time in Partial Differential equations, thanks to the course of lectures of Schwartz in TIFR on Complex Analytic Manifolds, especially on Hodge theory. I met in Paris the Japanese mathematician Takeshi Kotake who was visiting Paris to work with Schwartz and we collaborated on a work concerning linear elliptic operators with real analytic coefficients. During this period I studied the huge preprint of Kodaira and Spencer on deformations of complex structures and this turned out to be very fruitful in my future work. Schwartz gave me a copy of this paper. I have the impression that Schwartz himself was thinking on these matters in response to some questions of Weil on deformation of Riemann surfaces (and I guess that Schwartz was one of the "ellipticians" to whom Weil was referring to in his paper on this topic). The work of Kodaira and Spencer used crucially the theory of elliptic PDE with which I was familiar.

The great mathematicians in Paris were easily approachable by young mathematicians. I used to meet Schwartz regularly. I remember that Grothendieck spent some time explaining to me, on my request, his approach to deformation theory.

SR: Similarly, about the Harder–Narasimhan theorem which is now proving to have unexpected connections and applications to different areas of mathematics and also to Physics.

MSN: Once the moduli spaces were constructed the problem of computing numerical invariants of these spaces in particular Betti numbers arose. In the case of bundles of rank 2 with fixed determinant of odd degree, the Betti numbers were computed by P E Newstead by purely topological methods, using the description of these spaces in terms of unitary representations. Based on these results G Harder verified (in 1970) the Weil conjectures for this variety (in the case of a curve over finite field) at a time when Weil conjectures were not proved in general. P Deligne proved the Weil Conjecture in 1974. It was then natural to try to generalise the method of Harder and compute the Betti numbers of moduli spaces in the case of vector bundles of arbitrary rank and (coprime) degree, by calculating the number of rational points of the moduli space and using Weil Conjectures. Reinterpreting the fact that the Tamagawa number of SL(n) is 1, "Siegel's formula" gives an explicit expression (in terms of the zeta function of the curve) for the sum Σ 1/#Aut(E), the sum being over all vector bundles E; the sum over stable bundles gives essentially the number of rational points. To compute inductively the sum over unstable part (and hence the number of rational points), one uses a partition of this set by the "type" of the "Canonical filtration" of a vector bundle. Harder and I showed that any vector bundle has a unique filtration by subbundles such that the successive quotients are semi-stable and with their "slopes" (degree/rank) are strictly decreasing and used the type of this filtration, namely the degree and rank of the successive quotients, to partition the space. It turned out that this is a universal principle valid in several situations and enables one to endow an arbitrary object with a canonical filtration whose successive quotients are semi-stable objects.

SR: You spent almost all your career in TIFR till the age of 60. Can you please share your thoughts and experiences of those years?

MSN: First, the early days in TIFR, when a whole new world of mathematics was opening up to me as a young student having contacts with outstanding mathematicians, were perhaps the most exciting. I learnt, during my first two years, Functional analysis and Peter–Weyl theorem from Warren Ambrose, Algebraic Topology from Eilenberg (who, without ever mentioning the words "categories" or "functors", taught the whole course from a functorial viewpoint), and Complex Analytic Manifolds (in particular Kahler manifolds) from Laurent Schwartz, whose course paved the way for many of my future mathematical interests. There were intense discussions and joint seminars with fellow students. I remember a joint seminar with Seshadri on Weyl's book on Riemann surfaces, which turned out to be important later in our joint work. (There was no English translation of this book at that time.) Conversations with KC and KGR introduced me to various aspects of number theory and arithmetic groups. Bourbaki and Cartan seminars also played important roles in my early mathematical formation. The atmosphere and conditions for creative research in TIFR suited me perfectly. I could pursue my own independent directions of research, without any undue pressure to produce "results" quickly. The broad based interests, training and knowledge acquired in different fields and the excellent library in TIFR were of great help. The mathematician with whom I collaborated (in the fields of Algebraic and Differential Geometries) intensely and over long periods in TIFR was Ramanan. Our way of approach to and thinking about mathematics were very similar. I suppose I never worked so hard as during the period I was working with him.

I have had brilliant students in TIFR, who became eminent mathematicians and who have contributed to the renown of TIFR. I helped to create and develop schools of Algebraic Geometry, Differential Geometry and Lie Groups in TIFR.

During my stay at TIFR I also worked on creating structures and organisations for promoting Mathematical research in India.

SR: What about the years outside of India after that?

MSN: I went to the International Centre for Theoretical Physics at Trieste (ICTP) in 1993 as the Head of the mathematics section at the invitation of Abdus Salam. Actually he had invited me a few years earlier but at that time I could not accept the invitation. I was always interested in creating structures for promoting mathematical research in India and in developing Countries. In India as Chairman of the National Board of Higher Mathematical and internationally as member of EC of IMU and President of IMU's Commission on Development of Exchange, I had some experience in this direction. The aims of ICTP being to "advance scientific expertise in the developing world", working at ICTP provided a very good opportunity for promoting mathematics. I had complete freedom and financial resources at ICTP to set up schemes for this purpose. During my stay at ICTP many young mathematicians from developing countries have used the intellectual atmosphere and facilities at ICTP to establish themselves as leading mathematicians and in turn have built up mathematical research in their countries. After the stay at ICTP, I spent three fruitful years at SISSA (Trieste).

SR: How would you contrast your experiences of your career within and outside of India?

MSN: I enjoyed my work and career both in India and abroad; I had ample support from institutions in India and abroad for carrying out my personal research and for working for the development of mathematics. Working abroad at ICTP gave an opportunity to interact with young mathematicians from all over the world and help them in furthering their research. This, like my role in TIFR, gave me immense satisfaction.

SR: Were there any clear tipping points or turning points in your research career? Any "Eureka" moments?

MSN: I do not remember any "Eureka" moment, as such. But there were many exciting moments.

SR: Talk to us about a few of those …

MSN: There were really many. To name a few: The work with Ramanan on universal connections when things fell into place smoothly and swiftly and when we discovered the relationship between incidence correspondence in projective geometry (related to quadratic complex of lines) and the Hecke correspondence between moduli spaces of vector bundles on curves. Naturally also the work with Seshadri and Harder on stable bundles. A work which gave me much pleasure was the work with K Okamoto on concrete realisation of discrete series representations, especially as, when we started working on the question, I had hardly any experience in this field.

SR: Some aspects of geometry started out with connections to Physics and Hermann Weyl was one of the early visionaries to perceive these deep connections. What was your perception of these connections when you started out on your research career?

MSN: Hermann Weyl was a great hero of mine in mathematics; however when I was young I did not appreciate his visionary role in perceiving the deep connections between mathematics and physics. I began to read his writings, both technical and historical, in this area much later. It would be great if there are people like him now who can write about the interaction between mathematics and physics of the present day with profound knowledge of both the fields and with such authority.

SR: Starting from the 1960s, Algebraic and Differential Geometry forged ahead as Abstract or Pure mathematics. The links with String Theory, etc. were uncovered several decades after the mathematical advances were made. With the work of people like Hitchin, Witten, etc. the connections to Theoretical Physics were brought to the fore again. Today we seem to be in an era where the insights come from Theoretical Physics and the mathematicians are trying to catch up. What is your perspective on these intertwinings?

MSN: I started looking into some physics literature when I found that physicists were using some of my results with Ramanan and Sephardi by curiosity. I found that physicists had their insights (and discoveries) in certain mathematical problems, these insights apparently coming from some physical intuition. Examples in low dimensional topology, linear systems on moduli spaces and enumerative geometry, coming from gauge theory, conformal field theory, super symmetry come to mind. It seems that at present there are not so many remarkable insights coming from physics as it was a few years back. The major developments in the last few years (e.g. Fermat's theorem, Poincaré conjecture) come from internal dynamics in mathematics.

SR: Yet, the "internal dynamics" in the advance of these results are interesting. Fermat's theorem built on a vast body of earlier results in mathematics but from other areas, and in turn caused a surge in the area of arithmetic geometry, while Poincaré conjecture was proved using methods from within math, but in unexpected ways. What are your views on these remarkable interconnections within mathematics itself?

MSN: What fascinated me in mathematics is the exciting, amazing and often unexpected interconnections between various fields of mathematics and how this connection helps one to solve concrete problem in one of the fields. Who would have thought that Fermat's theorem would be related to the problem of modularity of elliptic curves over rationals, and this relationship would be a catalyst to attack the problem of modularity? As for the proof of the Poincaré's conjecture, one can say that it is a triumph of analysis combined with geometry. The deep techniques developed to solve the problem have been useful in solving other outstanding problems, which is a hallmark of a great work.

SR: What are your views on the state of Higher education and research in India?
A SWOT (Strength, Weakness, Opportunities and Threats) analysis.

MSN: Strength: We have some institutions of top level in undergraduate, graduate and doctoral education. Potentially very talented students.

Weakness: Mostly undergraduate education is weak. Many bright students do not want to pursue academic studies leading to original work.

Opportunities: Now seem plenty many higher education institutes being started. There are training programmes at various levels and substantial financial support to students.

Threats: Not having many qualified teachers and not too many people pursuing academic career, mainly due to internal and external brain drain.

SR: Are there any lessons we should be learning from the way things are done in other parts of the world? There is a change globally in the way research was done in the second half of the last decade, and now ... . Many Asian nations are emerging as forces to reckon with ... yet India seems to be far from making the Big Leap.

MSN: I do not know. We have availability of resources and generally support for development of mathematics in India. We have some first rate institutions of research and undergraduate training, though small in number for the size of the country. But the "internal and external brain drain" which I mentioned above, is a major constraint in making the Big Leap.

SR: Besides mathematics, what are your other interests or hobbies?

MSN: I am interested in literature, both Tamil and English (to a lesser extent French). I read quite a bit of detective fiction. Basically I am addicted to books. I like to listen to music, both carnatic and western.

SR: After such a long career in different aspects of mathematical research, what words of advice do you have for youngsters, especially those from India, who might want to embark on a research career?

MSN: I do not know if I have any special insightful advice. First of all get a broad based knowledge when you are a student, by reading good textbooks and seminar notes, classics and masters and by associating with good mathematicians. One often learns faster many branches of mathematics by discussions with teachers and fellow students. At the time of launching into research, one should be in an environment where good mathematics is cultivated, otherwise there is a danger of pursuing trivial research.

When you wish to learn a new subject or wish to pursue a new field of research, try to approach the field from as high and as sophisticated point of view that you are capable of.

SR: There are several areas of mathematical research where India has no presence. What are your thoughts on establishing a broader research base in mathematics in the country?

MSN: There are quite a few major areas in India where there is strength (Number theory, Lie groups and arithmetic groups, algebraic and differential geometry, algebra, analysis ...). We should concentrate on strengthening further these fields and increase the number of experts. At the same time, we could identify a few areas where we lack expertise and develop them drawing upon our experience during the past decades in cultivating the areas mentioned above.

SR: Thanks very much, Professor Narasimhan. It has been a pleasure interacting with you, and once again warmest wishes for now and the years ahead.

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