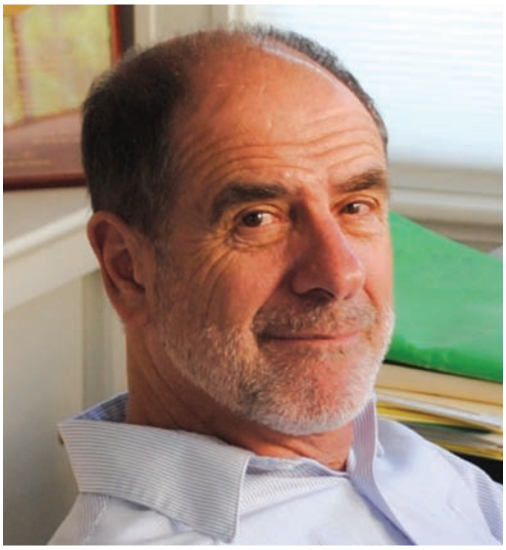
Two Evenings with B H Gross

Vijay Patankar and Sneha Patankar



Benedict H Gross

Professor Benedict H Gross is the George Vasmer Leverett Professor of Mathematics at Harvard University. He is a number theorist who has made fundamental contributions to many aspects of the subject. His most famous result is the Gross–Zagier formula, whose generalisations and higher dimensional analogues continue to fascinate mathematicians.

Professor Gross received his PhD from Harvard University in 1978. He joined Harvard University as a full professor in 1985 and since then has served as the Chair of Mathematics Department and as the Dean of Harvard College. In 1986 he was awarded a MacArthur Fellowship. He was awarded the Cole Prize of the American Mathematical Society in 1987 and was elected a member of the National Academy of Science in 2004.

Between January 7–10, 2013, Professor Gross gave a course of four lectures (Panorama lectures) titled "Some connections between representation theory and number theory" at the Tata Institute of Fundamental Research, Mumbai.

The following are excerpts of an interview with Professor Gross by Vijay M Patankar and Sneha V Patankar during his visit to Mumbai on sidelines of these lectures. He talks about his calling to Mathematics, his love for music and sports, and the importance of family.

Vijay M Patankar: Could you please tell us a bit about your childhood, any specific influences? How you became a mathematician?

Benedict H Gross: My parents were not particularly involved in mathematics. My father was a lawyer. My mom worked at home, and my grandparents were immigrants to the United States. They had not gone to college. I went through public school in a non-academic town.

And when I was 7 years old, that was the time the Russians sent out the satellite Sputnik around the world. The American leaders got very concerned that we were falling behind the Russians in areas such as space explorations, mathematics and sciences. So there were additional programmes introduced in the schools. The classes in mathematics and sciences in my public school were very good because they were trying to encourage kids to go into math and science. So I was very lucky. By 7th grade, when I was about 12, I was more advanced in math class. They pushed me ahead and by the time I got to the high school when I was 14 years old, I had exhausted the local high school curriculum. So they suggested to my parents that I go to a private school nearby where I could learn some more math and science, and I did. So by the time, I went to college, when I was 16, I had had a pretty good high-school education. Lucky me! And you know, I was interested in math competitions, the thing that kids do when they are that age. When I got to college, I went to Harvard and I realised that there were kids of my age who were well ahead of me. In my town, I was pretty good but compared to people who came from New York City or the people who came from Chicago, I wasn't that advanced. I started taking more advanced math courses and I just couldn't do it. I didn't have the background. So I decided, I would not go into math, but I would go into physics or chemistry. And I took a basic multi-variable calculus course in my first year. I was signing up to be a physics major which I liked very much. But then in my second year of college, I just happened to walk past a classroom where a great mathematician was teaching. It was Andrew Gleason, who was also a wonderful, wonderful teacher. I walked into the class and started listening to him, and after a semester, I was hooked. I was put in a trance by the way he presented the material and the way he thought about mathematics. He was teaching an advanced calculus course (about calculus of variations and differential forms). It was all challenging to me. But Gleason was very encouraging and I really thank him for persuading me to continue.

I did not know what it was like to be a mathematician. I had no idea what a mathematician did. In the newspapers, you can read about what physicists do and what chemists do, but not so much about what mathematicians do. The first great mathematician I met was Andrew Gleason when I took his course. That was absolutely inspirational. He persuaded me to do a major in mathematics. Because I was at Harvard, I had chances to take courses from some famous mathematicians. I did not know who they were but I took algebra from Richard Brauer and topology from Raoul Bott. So I was exposed to these wonderful wonderful mathematicians. I tried to a take a course from my eventual supervisor John Tate but I could not understand what he was doing. It was too advanced. In any case, by the time I finished college, I had terrific mathematical preparation. But I didn't yet have the motivation to go to graduate school. I wasn't really convinced it was my calling. And I really think in mathematics, it almost has to be your calling. It requires so much dedication, focus and concentration that if you just want to do it part-time, you should do something else. It is not a spectator sport. You have to really get into it. And I didn't know if I was ready for it, whether it was really the thing for me. I was interested in music and I had never travelled outside the US.

So when I was awarded a fellowship by Harvard to travel and I thought it would be better to travel to Asia because my dollar would go further. So I arranged to study music in Bali, Indonesia, where I studied Gamelan and in Chennai, India, where I studied the Carnatic violin from Mr V Thyagarajan. And that was a great experience for me. The year that I travelled, I brought a lot of math books with me and I read some great math books in the course of my travelling. By the end of the year, I was convinced that as much as I liked music, I did not want to do it as a professional, whereas I wanted to do mathematics as a professional. So, I ended up in England and I took a Masters degree (MSc) at Oxford. Just as I arrived in Oxford, Michael Atiyah arrived and gave a very inspirational course on Hilbert modular forms. If I ever needed any convincing to be a mathematician, that course did it.

VMP: Please tell us about the mathematics that you read during your travels and later while at Oxford, and how that has influenced you.

BHG: During that one year of traveling I read Serre's *A Course in Arithmetic* and Artin's *Geometric Algebra*. Those two books made a great impression on me. They were so elegantly written, and so coherent. I just wanted to read more. At Oxford, you have to do three areas for your MSc. So, I did Number Theory, Algebraic Topology and Complex Analysis. After the Masters, I thought it would be better for me to get a doctorate in the USA. So I applied to many graduate schools but I had been out for so long, I did not get into any of them. I did not get anywhere! So I wrote to Andrew Gleason and I asked him what to do. He said, I could come to Harvard as a special student and if I did well in the first semester then the faculty would consider admitting me. So I went as a special student and that is when I started my graduate studies. By that time I knew that I wanted to work in number theory. Not only that, I wanted to work with John Tate. I had read his introductory survey article on elliptic curves[a] which I absolutely loved. Before being published, this paper circulated as notes. In fact, when I was at Oxford I became friends with Andrew Wiles, an undergraduate student who was also interested in number theory. We looked at these notes together. Later, he went on to study at Cambridge to get his DPhil with John Coates. And I went back to Harvard to get my PhD with John Tate. I also had a lot of contact with Barry Mazur, who was doing his amazing work on the Eisenstein ideal. You couldn't ask for better training in number theory than to have the opportunity to go to Harvard then. It was fantastic.

VMP: On Serre and his influence — do you consider him as one of your mentors?

BHG: Yes! I was very lucky because the four years I was a graduate student at Harvard, Serre came to visit twice and gave courses in finite groups and analytic number theory. The opportunity to hear Serre speak, and to see how he thought about these things, was just invaluable.

Serre lectures so beautifully. I was attracted to mathematics by reading his books. Before I met him I thought — this is the way to do it. Then I went to one of his lectures and there was not a word out of place.

[a]John Tate, *Arithmetic of Elliptic Curves*, Inventiones Mathematicae, Vol. 23 (Springer, 1974), pp. 179–206.

Not a word out of place! And you thought to yourself, if I could do even a quarter as well as this it would be worthwhile.

In fact, Serre gave me my thesis problem. I had asked Tate for a thesis problem in my second year and he gave me his paper on p-adic divisible groups to read and suggested that I think about a general p-adic Hodge decomposition. But I didn't know how to do it. He then suggested I look for my own problem. When I was taking Serre's course on analytic number theory, he was talking about Hecke characters of imaginary quadratic fields and he showed us some examples. All the examples were for fields with class number one. I wanted to think about the general case, and that became my thesis on $\mathbb{Q}$-curves.

VMP: What is the first mathematical result that made a huge impression on you?

BHG: Oh! It is very elementary. In Gleason's course we proved that a continuous function on a compact set is uniformly continuous. I thought that was the greatest thing I had ever seen in my life. I was, like, Boy! Is that interesting! And I think it is always that for some people, you just see something and it is absolutely beautiful. Tate tells me that for him it was encountering quadratic reciprocity. You know for a lot of number theorists, once they see quadratic reciprocity, they just love it.

As a graduate student, I took a course with Tate on elliptic curves which expanded on the notes that Andrew and I had tried to read. That made a tremendous impression on me. Everything was just incredibly beautiful. Tate had a very original point of view on elliptic curves. You could really learn the subject from listening to him. But it was not any specific theorem. Reading *A Course in Arithmetic* by Serre, which gives a presentation of so many beautiful topics, also made a big impression.

VMP: Which is your favourite theorem (proved by you)?

BHG: The joint work I did with Zagier. That was an experience that happens once in a life time. Once in a life time! For the greatest mathematician, like Serre or Riemann, those things can happen three or four times because they are so extraordinary!

VMP: Could you please share a bit about what you learnt from John Tate; as an advisor, as a mathematician, as a human being?

BHG: He was a wonderful wonderful adviser, who had a great way of interacting with his students. I remember this so well, John would bump into you in the department and say, "Dick, I just understood something that you have explained to me many times before. I didn't really understand it before but I think I have understood it pretty well now. Let me show you how I think about it". Then he would go to the blackboard and show me the most beautiful thing in the world that I could have never thought of in a billion years. And by saying, "This is really just your idea, I am just trying to put it in my own words", he made you feel that you were part of the process. You didn't feel "What's the point in me doing this, he is going to understand it so much better than I am". Instead it was "Here is what we are thinking about together." So, I try to do that with my students. I don't say "You are going about this all wrong, this is the way I would do it". It is more like "Here is what you have shown me, let's just start it over and try it this way". In that way, everyone is involved. It is their idea anyhow, they are the ones who are working on it. Of course, it is a real pleasure to see someone like John who has such an organised mind work. It is as if all the impurities have been distilled out. He is also a very kind man, kind to everyone.

VMP: You have some phenomenal collaboration. Could you please tell us about it? How does it happen? About the dynamics of it?

BHG: I met Don Zagier in graduate school. Zagier came as a full visiting professor from Bonn. We immediately became good friends. We have a similar style of thinking about things and he also enjoys music. It was great to have that friendship, because a couple of years later when I started to think about the conjecture of Birch and Swinnerton–Dyer I realised that I need to make a difficult computation with Rankin L-series. I knew there was only one person in the world who could do it. I visited Don in Maryland, and he agreed to take a look at it, even though it didn't look very promising at the beginning. But since we were friends he agreed to try it. I think a lot of mathematics takes place as interaction between people who are personally friends. Sometimes you become friends through the mathematics. Sometimes you are already friends and you decide to do the mathematics together.

Mathematics in itself is a hard world, with no human aspect. Statements are either true or false. But the interactions you have with people when you work together are really enjoyable. A lot of my papers are in collaboration. A huge number of papers. And every time you collaborate with someone you learn so much. Zagier and I were in a perfect situation where we knew just enough in common to talk to each other. But that was it! He knew all this analysis of modular forms and I had the background on elliptic curves. We were able to learn a lot from each other. It was incredibly fun to work with him. And it was a great problem to think about.

I don't think one should ever be afraid to spend time computing a special case. Even the mathematicians in the past that we think as the most abstract thinkers, like Riemann, spent time computing. In Riemann's private papers historians discovered looooong computations of the first zero of the Riemann zeta function up to fifteen decimal places. Maybe he didn't publish it but that was the sort of thing he was doing at his desk. And I always tell my students that a good example is sometimes worth more than a string of lemmas. If you can't prove this in general, try to do it in some special cases. That will help build up the confidence necessary to do the next one.

I have other great collaborators. Among my colleagues, I had a wonderful collaboration with Mike Hopkins in homotopy theory and Lubin–Tate spaces and I have written papers with Curt McMullen on Salem numbers that I have really enjoyed. My friend and colleague Joe Harris and I have written a number of joint papers. Noam Elkies started as my student, but we have collaborated on a number of projects since. I had a long collaboration with Dipendra Prasad in representation theory, which Wee Teck Gan has recently joined. And in the past few years I have been collaborating with Manjul Bhargava on the arithmetic of hyperelliptic curves. I really enjoy working with others. I think it always starts with a question. You ask somebody a question. They know some things. They come back to you and after a lot of discussions you realise that you ought to write a joint paper about this.

Especially when you can find someone who you can talk to but where you have almost disjoint sets of knowledge. That is a lot of fun, and a good way to learn some new mathematics. So, I really think that my most interesting papers were all written in collaboration.

VMP: It seems that you like to teach.

BHG: I love to teach. There are places (like TIFR, IAS) where you don't have to teach. There are wonderful research people there who spend all their time thinking about new problems. Psychologically I am not of that type. I cannot think about research problems all day long. If you are not getting anywhere at least you have a schedule where at 10 o'clock you have to go teach a class. I find that very helpful. It is also fun that you try to communicate your enthusiasm to younger people. They give you a lot of energy in return.

And when you teach in a place like Harvard, the students are so strong. You know that certain people you are teaching are going to become the leaders in mathematics in the next generation. I mean, when you teach someone like Manjul Bhargava or Jacob Lurie as an undergraduate, you are just trying to pour as much out of yourself into them as humanly possible, because they are like sponges. Even when you teach students who are not going to go on as research mathematicians, you can give them a good last taste of the subject. So when they go into chemistry or financial math at least they have a nice memory that pure mathematics was a good thing and that they are going to use it.

VMP: Please share your thoughts on styles in mathematics, on sharing ideas, open-ness, on arXiv.

BHG: My feeling, Vijay, is that for great mathematics you don't need to put your name on it. It is so idiosyncratic. Things that are done by Zagier could only be done by Zagier, things that are done by Serre could only be done by Serre, those done by Deligne, only by Deligne. There is a certain style to it. When you sort of look at the paper you say, Oh, my God, I couldn't do that. And Manjul Bhargava has that style. He has his own way of approaching it. When he started, those of us who were working in elliptic curves were saying what is he doing, what is he doing? Then all of a sudden it became clear to us....

Many people are so worried this way or that way about priority, who discovered what, who did this, who did that. I don't worry that much. Some people hold their cards very close to their chest and don't want to say what they are working on. I think this is the stupidest thing in the world. I'm happy that other people know what I am working on — maybe they will be able to make progress where I won't. I feel that it is (not to be too philosophical) a group-mind that we participate in. We all benefit from the work and the insight of all the mathematicians in the past, nothing is lost. You know I spoke here on the work of Euler. You can't think about zeta functions without Euler. So we are the beneficiaries of that and we are lucky if we make a little contribution that someone else will use. But this marking out territory and calling it mine — I find that very strange.

I think arXiv is a terrific thing. And in Harvard, there has been a big effort for example to scan and put the entire library online. So that people don't have to make a trip and get into libraries to look into manuscripts — they are right there. And I think that that is the best thing that has happened with the Internet. The availability of knowledge, wikipedia, the arXiv so that you know someone in some small corner of the world can access it all. It is incredible. Whereas it used to be that there was this hierarchy of knowledge. You had to go the centre, you had to have permission and all that. So I am a big fan of the arXiv. I am sort of the older generation that isn't into blogging or putting up my thoughts. I would rather do it in a little more quiet way, give a talk, put the paper out there, let people read it.

VMP: Do you think that the Internet has made things worse in some sense?

BHG: That is a great question, Vijay. I was thinking about that because when I was working with Zagier we weren't together all the time. I would just wait two months and then I would write him a letter saying here is where I am now. And now the tendency, when you are working with someone, is "Oh, I got it. I got this teeny epsilon little idea, I will just send immediately by email". Right! And so you get all these emails back and forth and people spend too much time in email. It is too distracting. You know I get up in the morning and someone wants me to write a letter for something, and you spend an hour on the computer and you are mentally tired and you haven't had any time to think. And now with the Internet you have so much access to information. I can ask a question to anyone in the world and get a quick answer and you get instant gratification. But mathematics is not about instant gratification. And without the thought that goes between the emails they are not worth much.

I would like to slow that down but I am just as addicted to emails as everybody is!

VMP: What are your views on Nurture vs Nature?

BHG: I think that some people have enormous mathematical talent from birth. People like Zagier, people like Deligne are gifted beyond belief. They work very hard too, but they are just given a gift. So there is a certain amount of that but I think more of it is the Nurture, and the dedication, and the commitment. I see a lot of students come through and they are very very talented, but they are used to solving things in fifteen minutes. When you participate in competitions that is helpful and enjoyable. But when you face a really good math problem, it's a problem you can't solve in fifteen minutes. So those real math problems can be very frustrating. And that is when, I think you have to find in yourself the willingness to stick to something that takes much longer time span to solve.

The mental hardware is important but the hardwork is essential. Even for these enormously brilliant people, the ones that you see have succeeded are the ones that have really put their talent to work.

VMP: During the past few days I have observed that you have a "switch". Can you say a bit about that, about your working style?

BHG: Yeah. I go on and off. You know many mathematicians seem to be uncomfortable with the outside world. I get a real kick of being in the real world. I like a lot of sports, like tennis and windsurfing and skiing and golf. I like music and I like meeting people and I like socialising. But then I sit down and I throw the switch. And I really am very happy that I am able to spend several hours of a day; just sitting down and trying to think about something. I think that for all of our social interactions with mathematicians, in collaborations, in working at the black board, in going for lectures, you have to spend time at your desk. That is where you really come to understand things. And I really enjoy that time. I love that time, but I can't do it for 24 hours a day. And so I try every day to do some sport, or attend some concert, or do some this or that. So that when I sit down at the desk, I really want to be there. I don't feel like I am chained to it. Other people have different work environments. This is just what works for me. So very frequently when I am at home, I will get up early, go down to breakfast, sit down at the computer, sit down at my desk, work 2–3 hours. If I am getting somewhere, I keep going, if not, just get up and play a round of golf. I don't think anything is wrong with that. Whatever works. So I do have a switch. I have seen other mathematicians who are "ON" all the time. That is all they are thinking about. For me it does not work to think all the time. If I get in 3 good hours a day, that is enough.

VMP: You are deeply interested in music. You want to tell us a bit about that?

BHG: I have been playing music all my life. I have studied viola and a little bit of violin from the age of 6. I have always loved music and have played in a lot of orchestras and string quartets. I took music courses in college to learn a little bit of the theory. I still play once every other week in a string quartet of other mathematicians. I am not a very high quality musician but I love to listen to it and lose myself in it. I think there is a connection with mathematics. Nobody knows what it is but both are formal systems. My colleague Noam Elkies, is a wonderful pianist and composer. I don't play at that level, but I probably enjoy it just as much!

VMP: Could you please tell us a bit about family life?

BHG: My wife has always been super understanding, giving me a lot of quiet time when needed, and also my children, even when they were young. I believe that having kids and having a family is the most rewarding thing. Some people who are totally involved in mathematics miss out on all that.

VMP: Do you expect any significant progress in number theory, in mathematics in the next few years?

BHG: I have a standard answer for that. There are going to be tremendous advances in the next five to ten years, and all I can say is that they will be surprising to all. Both original and surprising. But when they are known, you will say, that was obviously the next thing to do. It is surprising at the time, but it is obvious in retrospect.

And all I can say is that in my own mathematical lifetime (which is about 40 years), just in my own field, I have seen several major advances. Not one of them you could have said was going to happen. Each one of them was like *WOW*!

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Sneha Salvi-Patankar was born in Ahmednagar. She completed her Masters in Mathematics from Pune. She is married to Vijay M Patankar. Her main interest lies in literature — English and Marathi. She enjoys travelling. She is currently a home-maker and is in the process of figuring out her professional interests.