Fields Medallist Maryam Mirzakhani (1977–2017)

Sameen Ahmed Khan

Maryam Mirzakhani, the first and to-date only woman to win the Fields Medal died on Friday, July 14, 2017. She had been battling with metastatic breast cancer since 2013. She was a professor at the Department of Mathematics, Stanford University, Stanford, USA. She was laid to rest in USA and a memorial service was held in her native Tehran, Iran.

Mirzakhani was just 40 years old! Her loss was grieved globally. Renowned mathematicians world-wide reacted to her death in such words, Peter Clive Sarnak, a mathematician at Princeton University and the Institute for Advanced Study said, "*her death is a big loss and shock to the mathematical community worldwide. She was in the midst of doing fantastic work. Not only did she solve many problems; in solving problems, she developed tools that are now the bread and butter of people working in the field*." The university president, Marc Tessier-Lavigne, said "*Mirzakhani's influence would live on in the "thousands of women she inspired" to pursue maths and science*." Iran's President Hassan Rouhani, who had congratulated her in 2014, released a statement expressing his great grief and sorrow: "*The unparalleled excellence of the creative scientist and humble person that echoed Iran's name in scientific circles around the world*," he further wrote, "*she was a turning point in introducing Iranian women and youth on their way to conquer the summits of pride and various international stages*."

The Fields Medal is a prize awarded to two, three, or four mathematicians under 40 years of age at the International Congress of the International Mathematical Union (IMU, http://www.mathunion.org/), a meeting that takes place every four years. The Fields Medal was established in 1936. Maryam Mirzakhani is the only woman amongst its 56 recipients. It is the most prestigious award in mathematics, often equated in status with the Nobel Prize. Mirzakhani received the Fields Medal in 2014 at the age of 37 for "her outstanding contributions to the dynamics and geometry of Riemann surfaces and their moduli spaces". She received the award during the International Congress of Mathematicians held in Seoul, Korea (13–21 August 2014, http://www.icm2014.org/). Maryam Mirzakhani is the first Iranian and the first Muslim to receive the Fields Medal. Her co-recipients are special in their own ways. Artur Avila the first South American, Manjul Bhargava the first person of Indian origins, Martin Hairer the first Austrian to be awarded the Fields Medal [1–3]. During the Seoul Congress, Subhash Khot (again of Indian origins) was awarded the 2014 Rolf Nevanlinna Prize by the International Mathematical Union, for his work related to the Unique Games Conjecture, as well as for posing the conjecture itself. This was the second time that the Nevanlinna Prize was awarded to an Indian; Madhu Sudan had won the Prize in 2002.

The Fields Medal (front): The head represents Archimedes facing right. The inscription reads: *Transire summ pectus mundoque potiri* (to transcend one's spirit and to take hold of or to master the world).

The Fields Medal (back): In the background, there is a representation of Archimedes' sphere being inscribed in a cylinder. The inscription reads: *Congregati ex toto orbe mathematici ob scripta insignia tribuere* (the mathematicians having congregated from the whole world awarded this medal because of outstanding writings).

As a youngster, Maryam Mirzakhani wanted to become a writer! When in high school, she developed a keen interest in solving mathematical problems and finding alternate proofs. She attended an all-girls high school in Tehran. As a teenager, Mirzakhani gained major international recognition by winning gold medals at the International Mathematics Olympiads held in Hong Kong (1994) and then at Toronto (1995). In the Toronto Olympiad, she notched a perfect score and another gold medal (https://www.imo-official.org/). In February 1998, a competition was held in the western city of Ahwaz, which brought together the mathematics community of the region. The bus transporting the participants from the Sharif University, Tehran, Iran, turned turtle and crashed into the ravines. The crash resulted in the deaths of two bus drivers along with seven award-winning mathematicians. Providentially, the survivors included Maryam Mirzakhani! A year later in 1999, Mirzakhani received the BS degree from the Sharif University of Technology, Tehran, Iran. Then she moved to Harvard University, USA to pursue her PhD under the guidance of the world-renowned Fields Medalist, Curtis Tracy McMullen. Her stay at Harvard was marked by her extraordinary determination and relentless questioning. Although she had a serious language barrier, it did not hinder her from peppering her professors with numerous questions in English. She noted the responses of her professors in her native language Farsi (Persian). She obtained her PhD in 2004. Mirzakhani had an Erdős number of three. The collaboration path is: Maryam Mirzakhani coauthored with Ebadollah S. Mahmoodian coauthored with Mehdi Behzad coauthored with Paul Erdős.

Maryam Mirzakhani was an exceedingly original mathematician, who made a host of striking contributions to geometry and dynamical systems. Her work connects several mathematical disciplines including hyperbolic geometry, complex analysis, topology, and dynamics. Mirzakhani gained widespread recognition for her early results in hyperbolic geometry, particularly on a problem known as the *prime number theorem for simple closed geodesics*. Her approach led to a new proof of a conjecture that had been made by string theorist Edward Witten (1990 Fields Medalist). Witten's conjecture is a crucial result in string theory. The conjecture was first proved in 1992 by Maxim Lvovich Kontsevich of the Institut des Hautes Études Scientifiques, Paris, France. Kontsevich was awarded a Fields Medal in 1998 in part for this proof. Mirzakhani provided a new proof of the Witten's conjecture using an unexpected mathematical machinery. This led her to the study of dynamical systems associated with spaces of Riemann surfaces. It is also in this field, that she and her collaborators made fundamental breakthroughs.

Mirzakhani did her PhD in 2004 at Harvard University, under the supervision of the Fields Medalist Curtis Tracy McMullen. Her PhD thesis was a masterpiece. In her thesis, Mirzakhani solved two longstanding problems. "Either solution would have been newsworthy in its own right", according to Benson Stanley Farb, a mathematician at the University of Chicago, but then Mirzakhani connected the two

into a thesis described as "truly spectacular." Mirzakhani's thesis resulted in three single-author papers published in the three top journals of mathematics: *Annals of Mathematics* [4], *Inventiones Mathematicae* [5], and *Journal of the American Mathematical Society* [6]. "The majority of mathematicians will never produce something as good," Farb said "and that's what she did in her thesis."

Maryam Mirzakhani had, along with University of Chicago mathematician Alex Eskin and University of California, San Diego, mathematician Amir Mohammadi, written monumental papers culminating in a *magic wand theorem*, not about individual hyperbolic surfaces but about whole spaces of them. Their work has applications to the classical physics problem of understanding the motion of a billiard ball as it bounces around a polygonal table and constitutes one of the most sought-after advances in the area known as Teichmüller dynamics [7]. The rigidity theorems, she proved have numerous and far-reaching applications. That investigation of this seemingly simple action of a billiard ball has led to a 200-page paper which, when it was published in 2013, was hailed as "the beginning of a new era" in mathematics and "a titanic work." [8]. A shorter and expository version of this long paper was published by Anton Zorich [9].

Maryam Mirzakhani was a Clay Mathematics Institute Research Fellow and an assistant professor at Princeton University, from 2004 to 2008. In 2008, Mirzakhani joined the faculty of Stanford University, as a professor of mathematics and held this position until her death. Experts say that her achievements "combined superb problem-solving ability, ambitious mathematical vision and fluency in many disciplines, which is unusual in the modern era, when considerable specialisation is often required to reach the frontier". Her honours include the 2009 Blumenthal Award for the Advancement of Research in Pure Mathematics and the 2013 Satter Prize of the American Mathematical Society. The Stanford University organised a memorial service on October 23, 2017. Mirzakhani did not live long enough to collect other awards — six of the 18 *Abel Prize* laureates are Fields Medalists (http://www.abelprize.no/) and five of the ten *King Faisal International Prize* winners in Mathematics are Fields Medalists (http://kfip.org/, see Ref. [10–11] for details). Unlike the Fields Medal, both of these prizes do not have any age limit. She would have been a prime candidate for the *Mustafa Prize for Sciences* launched by her native Iran in 2015 [12].

Mirzakhani's contributions inspired thousands of women to pursue mathematics and science. Her legacy will continue to inspire young girls and boys from all walks of life the world over. The faculty of mathematics, Sharif University, Tehran, Iran, where she studied is being renamed as *Mirzakhani*. Mirzakhani is survived by her husband Jan Vondrák, and their daughter Anahita as well as her parents, sister and two brothers. Vondrák originates from the Czech Republic. He is a theoretical computer scientist, applied mathematician, and an associate professor at Stanford University.

Bibliography:

1. De Melo, W., Poonen, B., Quastel, J. and Notic, A.Z., The Work of the 2014 Fields Medalists, *Notices of the American Mathematical Society*, **62** (11), 1334-1349 (2014). http://www.ams.org/notices/201511/rnoti-p1334.pdf

2. Kronzek, R., International Congress of Mathematicians, *Asia Pacific Mathematics Newsletter*, **4** (4), 14-16 (October 2014). http://www.asiapacific-mathnews.com/04/preserved-docs/0404/0014_0016.pdf

3. Alladi, K., Manjul Bhargava's Fields Medal and Beyond, *Asia Pacific Mathematics Newsletter*, **4** (4), 17-20 (October 2014). http://www.asiapacific-mathnews.com/04/preserved-docs/0404/0014_0016.pdf

4. Mirzakhani, M., Growth of the number of simple closed geodesics on hyperbolic surfaces, *Annals of Math.*, **168**, 97-125 (2008). http://dx.doi.org/10.4007/annals.2008.168.97

5. Mirzakhani, M., Simple geodesics and Weil-Petersson volumes of moduli spaces of bordered Riemann surfaces, *Inventiones mathematicae*, **167** (1), 179-222 (2007). http://dx.doi.org/10.1007/s00222-006-0013-2

6. Mirzakhani, M., Weil-Petersson volumes and intersection theory on the moduli spaces of curves, *J. Amer. Math. Soc.*, **20**, 1-23 (2007). http://dx.doi.org/10.1090/S0894-0347-06-00526-1

7. Eskin, A., Mirzakhani, M. and Mohammadi, A., Isolation, equidistribution, and orbit closures for the $SL(2,R)$ action on moduli space, *Annals of Mathematics*, **182** (2), 673-721 (2015). http://dx.doi.org/10.4007/annals.2015.182.2.7

8. Eskin, A. and Mirzakhani, M., Invariant and stationary measures for the $SL(2,R)$ action on moduli space, *E-Print arXiv*, arXiv:1302.3320 [math.DS]; 204 pages, (2013). https://arxiv.org/abs/1302.3320

9. Zorich, A., Le théorème de la baguette magique de A. Eskin et M. Mirzakhani, *Gazette des Mathématiciens*, **142**, 39-54 (2014). http://smf4.emath.fr/Publications/Gazette/2014/142/smf_gazette_142_39-54.pdf; Anton Zorich, The Magic Wand Theorem of A. Eskin and M. Mirzakhani, *E-Print arXiv*, arXiv:1502.05654 [math.DS]; https://arxiv.org/abs/1502.05654

10. Khan, S. A., 2014 King Faisal International Prize Goes to Gerd Faltings, *Asia Pacific Mathematics Newsletter*, **4** (1), 26-27 (January 2014). http://www.asiapacific-mathnews.com/04/preserved-docs/0401/0026_0027.pdf

11. Khan, S. A., 2014 King Faisal International Prize for Science and Medicine, *Current Science*, **106** (4), 500 (2014). http://www.currentscience.ac.in/Volumes/106/04/0500.pdf

12. Khan, S. A., Iran Launches the Mustafa Prize for Sciences, *Current Science*, 110 (6), 961 (25 March 2016). http://www.currentscience.ac.in/Volumes/110/06/0961.pdf

Sameen Ahmed Khan
Department of Mathematics and Sciences,
College of Arts and Applied Sciences,
Dhofar University,
Salalah, Sultanate of Oman
rohelakhan@yahoo.com
http://orcid.org/0000-0003-1264-2302