Interview with Gus Lehrer

January 9, 2012, University of Melbourne

Peter Hall

Gustav Lehrer

"I am firmly convinced that mathematical thinking can be taught, just like reading and writing. Of course, not everyone who learns English will be able to write like William Shakespeare, and likewise not everyone will be able to do research in mathematics. But people need to appreciate that they actually do mathematical thinking every day, mostly without realising it."

Introduction: Gustav Isaac Lehrer was born in Munich, Germany, on January 18, 1947, and migrated to Sydney, Australia, with his parents at the age of three. He is an algebraist at the University of Sydney, where he has worked for most of his career since returning to Australia from the UK, in 1974, after his PhD and postdoctoral work. From mid 1996 to mid 1998, he was the Director of the Centre for Mathematics and its Applications at the Australian National University.

Gus is particularly well known for developing, with Bob Howlett, a branch of representation theory known today as Howlett–Lehrer theory, which has found application in several different areas of mathematics. Another highlight of his work is his study of the geometry of configurations of points, using algebraic geometry to relate continuous and discrete approaches to the problem. With his former student, John Graham, he also invented cellular algebras, which are now used in the theory of quantum groups and related topics, and form a link between mathematics and physics. These very considerable research contributions, and others, led to his election to the Australian Academy of Science in 1998.

In Australia, Gus is also well known for his efforts to maintain high standards in research in pure mathematics. In this connection, among others, his leadership in developing international linkages is widely appreciated. He served a term on the mathematics grant-awarding panel of the Australian Research Council (ARC), the main Australian research granting agency, and is well placed to comment on their activities in mathematics. He is also keen to make the wider community aware of the advantages of mathematical thinking. In this regard the quotation at the head of this interview is pertinent; it came from Gus during this interview.

Peter Hall: Thank you, Gus, for taking time out for this interview. I'd like to begin by going back to your very early life, and especially the lives of your mother and father, who must have been affected profoundly by the wartime horrors of Europe.

Gus Lehrer: My parents were both survivors of the Holocaust. My father, who had been born in Stryj, in the region of Gallicia in south-east Poland (now in Ukraine), hid from the Nazis for 14 months, in an attic in Stryj. In fact, this is how he met his future wife, who was also brought to hide there. However, not many Jews survived the Nazi occupation, and my mother and father lost all their family members in the Holocaust.

It should be remembered that Polish Jews, such as my mother and father, had been suffering under repressive laws for a considerable period prior to the war. In particular, they were effectively not allowed to own land or have government jobs. This led to an attitude of keeping your head down, and not seeking fame or glory, which I inherited from my parents.

Immediately after the war, my parents were in Germany as "Displaced Persons". At the time of my birth, my father had TB, my mother had typhoid fever, and they contemplated giving me up for adoption, because they feared they could not look after me. However, things improved, and the three of us arrived in Fremantle, Australia, on Melbourne Cup Day in 1950. My parents initially wanted to migrate to the USA, but my father's history of TB made this impossible. We had a sponsor from Sydney, who had wisely come to Australia from Poland in 1937, so we continued on the boat until it reached Sydney. Nevertheless, my first memory of Australia is from Fremantle, and was particularly auspicious: When we disembarked there, a woman walked up to me out of the crowd, and gave me an ice-cream.

My father had had a brother, executed by the Nazis in 1942, who had completed medical studies in Bologna because of the Polish "numerus clausus" policy, under which Jews were not permitted to study many subjects at Polish universities. My father would probably also have done medicine, but because of the intervention of the war, he was taught by the Russians during their occupation of Poland how to run a shoe factory. He had no experience of shoe-making, but in wartime the Russians had a great need of shoes. His parents had been business people, and with this background, and his experience running a shoe factory, he took the first available opportunity to go into business in Australia. (For almost the first two years he worked here on a telephone assembly line.) He eventually built the business into a successful textile manufacturing operation.

My mother came from a musical family and had studied opera singing in Lvov (then in Poland, now in Ukraine). She was very accomplished, and earned most of the family income in Germany by singing Schubert Lieder on Süd-West Rundfunk in Germany between 1945 and 1947. When my father started in business, she worked for several years in garment making, leading a small team of workers, and often spending 16 hours a day at the machines.

My family's friends in Sydney had mostly been members of left-wing Jewish youth groups in Europe, before migrating to Australia. They didn't provide us with much material help, but their sponsorship of our family, which meant that they undertook to take responsibility for us in the event of misfortune, was critical.

In 1957 and 1960 my two sisters were born. My mother became a carer for them, and took up singing again. She was on the Elizabethan Theatre Trust for many years. Both sisters completed arts degrees, both have three children, and both inherited what I might call my father's "imaginative approach to life"— Elisabeth, the older sister, went on to take a course in acupuncture, and Carolyn is a successful sculptor. Earlier, Elisabeth taught English and History at school. Carolyn started her family soon after finishing her degree.

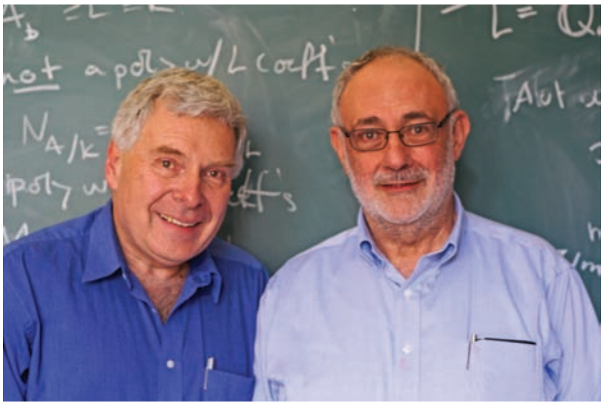
PH: Your family was uprooted by the war, and moved to the other side of the world. It must have been especially traumatic for you as a child. Can you tell us something of your early life?

GL: After a brief period in a migrant hostel, our family managed to rent an apartment in the Sydney suburb of Maroubra. I was first sent off to a boarding school at the age of five. It was a rather brutal institution where the whole day was spent idly rolling tyres around and playing. Luckily I could read German, so I was able to pick up reading and writing essentially without tuition. A year later, I was sent to Maroubra Bay Public School, of which I have mixed memories. In the 1950s, Australia was not a very hospitable place for "refos" ("refugees"), and although there were some other refugees at the school, on the whole, we were not well treated, with some teachers being notable exceptions. I recall new immigrant children being ridiculed by the class, led by the teacher, for their poor accent, when they had recently arrived in the country.

The wartime experiences of my family, and the challenges that they faced in Australia, had inevitably left me with a feeling of insecurity, or lack of self confidence. For all these reasons I was a very poor student at primary school, even to the point of playing truant often. I was a good friend of the boy at the bottom of the class. However, because IQ tests were still being used to allocate students to classes, I was always placed in the top stream, and in the 6th (and last) primary school grade, I had my first positive intellectual experience with an enlightened teacher who showed some understanding of my social problems.

I found myself selected to go to Sydney Boys High School, where I was placed in the top class. This was a revelation for me, because at that time there was a very hierarchical selectivity, and Sydney High took the best students from a very large catchment area. I recall that in the final school exams, 19 of our top class of 27 were in the top 100 in the state. The quality and nature of the teaching was something quite new to me, and it was there that I discovered I had a special affinity for mathematics. I had two particularly inspirational teachers, Geoff Ball, later a colleague at Sydney University, and John Harrison.

PH: You recovered remarkably well from the challenges of your early years at school. Did you go



Peter Hall and Gustav Lehrer

directly from Sydney Boys High to university?

GL: Yes, I entered Sydney University, in the Faculty of Medicine. I was just 16, and mathematics had not yet occurred to me as a possible career. However, at that time I could do precisely the subjects I would have done if I had enrolled in Science with a view to studying mathematics. In my first year I went to lectures by T G Room, which I now realise were somewhat eclectic, disorganised and flawed in other ways. Nonetheless I found them inspiring; he covered many varied topics, including projective geometry, mathematical logic, spherical trigonometry, and topics in algebra and number theory. It was the sort of course which would today be given very poor ratings by students, but in my opinion is sorely missed. (Economic rationalism does not always lead to the optimal outcome in undergraduate teaching.) At the end of first year, although I had to forgo a prize for medicine, I decided to continue with mathematics in the faculty of science. It was in second year, when the courses were much more systematic, that I first had the inspiration that I might be able to organise and explain some material at least as well as the text books and lecturers; this led to the idea that mathematics might be a career option. I was influenced over the next three years by several of the mathematicians at Sydney University: Tim Wall, who taught me what abstract algebra really was, and gave some inspiring insights into Galois theory and algebraic number theory; Don Barnes, John Mack, and Bill Smith-White for his exceptionally lucid lectures on analysis, to which I was always attracted, even though it is not my speciality.

PH: We had all those lecturers in common, although I was about six years behind you. Did you go straight into graduate work after finishing at Sydney?

GL: Yes, after my Honours year I was awarded a DAAD (Deutscher Akademischer Austauschdienst) scholarship to Tübingen to study with H Wielandt, and also a Commonwealth Scholarship to Warwick, to study with J A (Sandy) Green. I decided to take up the latter.

At Warwick, I entered a completely new world in 1968. Warwick was at the time new, having been founded in the early 1960s, but it had very entrepreneurial leadership from its Vice-Chancellor, (later Lord) Butterworth. He had managed to get Green and E C (Christopher) Zeeman to head up the new Mathematical Institute. They were undoubtedly among the top three algebraists (respectively, topologists) in the UK, and by the time I arrived at Warwick there were 70 graduate students in pure mathematics, the largest pool in Europe. The students came from all over the world, and the institute ran a series of one-year symposia on various subjects, during which there was a Nuffield Professor appointed.

Stephen Smale was the Nuffield professor when I arrived at Warwick. I remember particularly a year on algebraic geometry, during which David Mumford was the Nuffield Professor. I was influenced considerably by him in my studies of representation theory, in that he initiated discussions with me about potential links to geometry of my research project on the character theory of the special linear groups. The basic reference for my thesis was a seminal paper of Green on the general linear group, dating back to 1954. It was then regarded as a nice piece of work appreciated by specialists; it is now recognised as one of the masterpieces of the last century. People do not generally realise that I G Macdonald (who incidentally was my PhD examiner) wrote his famous book on symmetric functions because of Green's paper, where "Green polynomials" were defined.

As well as Mumford and Green, I was influenced by a statement made by the Oxford mathematician G Higman, who said that "the representation theory of the general linear groups must be rewritten by each generation in the idiom of the day". This made me realise that there are certain mathematical themes which are "ubiquitous", and it is a search for these that has guided my interests throughout my career. Thus, although I have not been much concerned with practical applications, I have always paid attention to the range of applicability of a set of ideas. For example, permutations occur everywhere, so the representation theory of the symmetric groups is "fundamental"; similarly, linear transformations and the general linear group; and again, spaces of configurations of distinct points occur everywhere (and are linked to the above topics).

I finished my thesis on the special linear groups (matrices of determinant one) early in 1971. In it, I gave one of the early expositions of Harish-Chandra theory for a reductive group over a finite field. However I did not solve the problem completely, and am proud to report that it is still open to this day, not having yielded to the beautiful geometric methods developed for the general case, because there are significant arithmetical complications, which have not been completely solved, although the "character sheaves" of Lusztig make inroads to the problem.

PH: The mathematics research environment at Warwick was obviously very unlike the much more measured one in Australia in those days. Was it all work?

GL: No, not quite! I played squash at county level, and I met my wife Nanna while I was a graduate student there. She is from Norway. After my PhD I took a "Postdoc" (then called a Junior Lectureship) at Warwick for one year, and a permanent job at Birmingham University, to wait for her to finish her course in physiotherapy. We married in 1974, on my return to Australia to a job at the University of Sydney, and we have three children and four grandchildren. Our eldest, Lisa, did a PhD at UBC (Canada) in mathematical logic, and works in the area of non-profit organisations and public health policy. The next in age, Alex, did an honours degree in chemical engineering, followed by a CFA (Chartered Financial Analyst) correspondence degree. He now runs the family businesses. Our youngest, Eddie, did Economics, and works at Macquarie Bank.

I should add that the general UK mathematics environment at that time was not particularly encouraging. For example, the position I took at Birmingham was the only permanent job advertised, in pure mathematics in a UK university, during that year (1972–1973). Warwick was something of a mathematical oasis in the UK at that time.

PH: Please tell us a little more of your mathematical life. How have the opportunities changed for Australian mathematicians since your return to Sydney?

GL: It is a great irony that, in some sense, mathematics has never been healthier than now in Australia, although there are a great many challenges facing a young person commencing a career here. Today there are many Australian mathematicians at top institutions around the world: Harvard, Oxford, Cambridge (UK), MIT, Stanford, Caltech, UCLA; and we have recently had our first Fields Medallist (Terry Tao). There are now numerous opportunities for young Australians to win research fellowships which relieve them of practical duties for several years, etc. Further, there are many opportunities for travel and to invite overseas visitors to Australia.

However, the general environment in which young mathematicians work is much worse than when I began as a lecturer in mathematics at Sydney University in 1974. The standard workload is much higher, particularly in view of the huge bureaucratic demands on all academics, and generally students come to university with much less preparation than previously. This is part of the "crisis" which has been highlighted in successive reviews of Australian mathematics. One of the perceptions which still seems to permeate the public's thinking about mathematics is that it is not for everybody; this is one of the greatest challenges we face.

Although Australia's mathematical isolation has been reduced, there is still a danger that the value system applied to mathematical work in Australia is out of step with the world at large. This is partly because of our method of evaluating research, both by the ARC and elsewhere, involves many assessments by people without special expertise in the area concerned, principally because there is not enough expertise available here. This leads to the danger that subjects with low entry barriers will be more strongly promoted here than in a more competitive environment.

An example is the relatively low representation of Australians in the rich field of algebraic geometry, which has flourished for the past 60 years, and is now enjoying renewed vigour through its connections with mathematical physics. This field, and topology, are under-represented in this country. Because of our small size, serious Australian mathematicians inevitably will be measuring themselves by the best international standards, and that means those at the best institutions, such as the ones mentioned above.

I do believe that the primary focus in determining whether or not a certain mathematical enterprise should be supported should be on quality. Mathematics is about fundamental principles, their interaction, and applicability. In my experience at the ARC, and elsewhere, I have found that sometimes we are seduced by the false god of "applicability". One of my favourite tests of the importance of a mathematical subject is its universality. That is, does the question arise in a variety of different contexts? This is what I believe to be the type of applicability we should look for, rather than a more narrow view, based on traditional divisions into "pure" or "applied" mathematics.

PH: Perhaps we can move the focus more sharply to your own contributions. Tell us something of the highlights of your own research career.

GL: My research is in representation theory, which is a mathematical encoding of symmetry. Since symmetry is possibly the most important organising factor we use to understand the world, this field passes my "universality" test. Moreover, it may be studied in the context of different mathematical specialities: algebra, analysis, topology, geometry. Many people know that there are just three ways of covering the plane with regular polygons: triangles, squares and hexagons. A smaller number of people know that there are just five regular (real) polyhedra: the tetrahedron, cube, octahedron, icosahedron and duodecahedron. This reflects the fact that the "universe" permits only certain types of symmetry. The Lie groups, which are highly symmetrical "manifolds", have been completely classified, and my research has been in areas of algebra, topology and geometry which relate to how these could conceivably appear in a context different from the one where they arise. That is, how can a Lie group be "represented"? When I began at Warwick there was an intense hunt on to find all the finite simple groups. Almost all of these are Lie groups over finite fields, but there is a finite number of "exceptional" ones, which were discovered over 50 years (one of them by Janko, in Melbourne in the early 1960s). There was intense interest in the problem of classifying all representations of the finite Lie groups, and my thesis was about one series of them.

There had been developments in the "continuous" theory by Harish-Chandra from Princeton, and one of the guiding principles of the new algebraic geometry of Grothendieck is that such theories should be context free, so that the continuous and discrete theories should be essentially the same. So Tonny Springer (who, sadly, passed away last December) had adapted the Harish-Chandra theory to the finite field case, and come up with the notion of "cuspidal representations" (Harish-Chandra's concept in the continuous case), and with the "decomposition problem" for induced cuspidal representations, which was explained to me by Steinberg in 1972 at Warwick.

After my return to Australia, Green wrote to me saying that one of his former students had a student in Adelaide who was working on these things. This turned out to be Bob Howlett, and together we solved Springer's decomposition problem, and invented what is now known as "Howlett–Lehrer theory". This is now applied in several different areas of mathematics. It is used for decomposing geometric objects called perverse sheaves, all wildly beyond what we had in mind when the work was done. The basic idea was to reduce a complicated problem (decomposition) to something known (the theory of Hecke algebras). This was the highlight of my career to that point. The general problem of constructing representations (in particular cuspidal ones) was solved by Deligne and Lusztig in their famous 1976 paper on finite reductive groups. This used the realisation of these groups in the context of algebraic geometry to construct geometrically spaces upon which they act. It is interesting that to this day, Howlett–Lehrer theory is still referred to regularly, and built upon.

I have continued to think about problems in algebra, geometry and topology which arise from this fundamental context: What are all possible situations with given symmetry properties?

The next highlight was again related to the general guiding principle of using algebraic geometry to relate the continuous to the discrete. An example of this principle is that the geometry of the complex solutions to the equation $x^2+y^2 = 1$ should bear some relationship to the solutions of this equation in congruences modulo a prime number. In the process of studying some reflection group representations, I came upon the problem of determining the geometry of the space of configurations of n distinct points in the complex plane, encouraged by Lou Solomon of the University of Wisconsin at Madison. This problem has connections with knot theory, and is responsible for my interest in that subject. To my surprise, I found that this was an unsolved problem in topology, and I realised that algebraic geometric methods could be a key to the solution. Over the next 15 years I developed several approaches: analytic (using differential forms), topological (cohomology) and algebraic geometric, and this has been a very rich vein of research for me. I have collaborated with Mark Kisin on arithmetical aspects of the general theory, and with Alex Dimca on analytical geometric aspects.

The other specific highlight I wish to mention is the invention of cellular algebras, with my former student, John Graham. This theory provides a means of "deforming" structures which split, to more complicated ones, which do not. It is a subject which has been taken up in many centres, but I am extremely gratified that China is a great centre for the study of cellular algebras. Although it has probably not contributed much to Australia's balance of payments, I daresay that cellular algebras are among our successful exports to China! They are now used in the theory of quantum groups and other areas, and thus form a link between mathematics and physics. I have just spent several weeks at the Institut Henri Poincaré in Paris with a group of physicists and mathematicians, discussing "spin chains" with the aid of cellular algebras.

I cannot comment upon my research without saying how much fun I have had, and in particular, what an honour it has been to collaborate with fantastic people such as those mentioned above.

PH: That's fascinating, Gus. I wonder whether we could set these comments against a more general background, of how algebraic research has evolved during your career, and consider where it is going today.

GL: Mathematics is, like all other fields of human endeavour, subject to fashions. In the 1950s it was quite acceptable to write papers proving obscure results of a very abstract nature, such as if a certain algebraic structure satisfies a certain (often large) set of properties, then it must belong to a specific (short) list of (known) structures. This period of extreme abstraction was spawned partly by the success of algebraic topology and algebraic number theory, and partly by the discovery of new simple groups through such "characterisation" results. But the balance swung too far, and over the first 10–15 years of my career there was a move towards papers discussing interesting examples — the opposite extreme of the totally abstract papers of the previous period.

However, the fact that some of the most spectacular advances in representation theory were made by establishing very abstract "equivalences of categories" moved the pendulum back. For example, the Verma conjecture was a very specific statement about multiplicities of certain modules in the class of Verma modules; it stated what certain positive integers should be. It was first proved in the 1980s by Beilinson, Bernstein, Brylinsky and Kashiwara, by showing that a certain category of modules is equivalent to a totally different category defined in terms of differential equations on a certain manifold.

This led to two great trends. The first was a geometrisation of representation theory. This is characterised by the search for a geometric context whenever one does representation theory; that is, one looks for similarities in structure between sets of representations, and sets of sheaves (for example) on certain algebraic manifolds. The second is a wider opening of opportunities for cross-fertilisation between subjects which ostensibly have no connection with each other. These two trends have of course had the effect of raising the entry barriers to the subject. However, I believe that both characterise to some extent the development of the whole of mathematics in the last 30 years. To seriously study singularities, which previously required only a knowledge of differential analysis, now requires microlocal analysis, which involves derived categories, and some very sophisticated algebra.

There have been two further paradigm shifts in representation theory over the last 15 years. First, the notion of a deformation, which could be interpreted in terms of "non-commutative geometry", has been enormously influential through the study of quantum groups, which have also been used to solve some fundamental problems about multiplicities. Since these were invented by the Leningrad school of mathematical physicists to study physical problems, this has created profound links between the two areas. Additionally, the wheel has turned full circle in the trend away from abstraction; a favourite word in representation theory is "categorification". This is a principle rather than a theory, but it says roughly that positive integers should be interpreted as dimensions, that relationships between numbers should be arrows in a category, and that maps should be functors. It is interesting that the concept was invented by Khovanov, in the context of providing a structure to "explain" the Jones polynomial invariant of a knot or oriented link.

In summary, there is a myriad of new ideas, and of interactions with many and varied areas of mathematics. The subject is still vibrant, with many young players, and a Fields Medal awarded in 2010. One reason is that some of the basic, easily stated problems remain, such as, what are the dimensions of the irreducible representations of the symmetric groups over the field of two elements? There is no shortage of motivation for young guns!

PH: Could you tell us a little about your students?

GL: I have had many students; they are the lifeblood of any mathematical career, and my students work today in many different places. For example, Matthew Dyer is a Professor at Notre Dame (Indiana), and I still collaborate with him to this day. He is probably the world leader in Coxeter groups and Kazhdan–Lusztig theory. Leanne Rylands is an Associate Professor at the University of Western Sydney, and has been very influential there. John Graham, a brilliant individual, has worked in the finance industry for about 10 years, as has Jerome Blair. John retains an interest in mathematics, and still publishes from time to time. Ian Grojnowski is a Professor at Cambridge, in the UK, and has done beautiful work in "geometric Satake". Anthony Henderson, who did a PhD with George Lusztig at MIT, has returned to Sydney; last year he won the inaugural Heyde Medal in pure mathematics, awarded by the Australian Academy of Science. My *cotutelle* student, Emmanuel Letellier, is *Maître de conférences* at Cäen. In 2011, I had two students submit PhD theses. They were very different; one (Jon Kusilek) is already working in banking, while the other (Justin Koonin) is not sure what is next. I maintain contact with most of my students, and I am very grateful for the enrichment they have brought to my life.

PH: Perhaps we could conclude with a little advice for young men and women starting today.

GL: I believe that a career in mathematical research can be driven only by an irresistable desire to understand and discover. This is very different from a career in mathematics, which I would recommend to anyone who likes it. These days, it seems clear that if one embarks on a potential career as a mathematical researcher, there are many ways to opt out, because many employers recognise that the skills which go to make a successful mathematician are very adaptable to different contexts. This means that the risks associated with embarking on a career in research are somewhat mitigated.

Mathematics is a subject with a universal perspective. All serious mathematical research is done in the context of the whole world, because one is pushing the frontiers; this is in distinction to using mathematics, which is necessary in many contexts, and is more "local" in its focus. Therefore, my main advice to a young person contemplating a career in mathematics is simply to "follow your dreams". Secondary advice would include exhortations to read the masters, and to always try to be where the great advances are being made. However, do not follow fashion slavishly; rather, let your own informed curiosity determine where you direct your efforts.

Never be afraid of looking silly by asking questions. Silly questions have led to some of the most original ideas in mathematics. I would also advise starting students not to be afraid of collaboration. The internet and email have tempered the tyranny of distance somewhat, but isolation is still an ever-present danger, particularly in Australia. Collaboration with international partners is a very good way of forcing yourself to keep up with what is going on everywhere.

One final word... With the advent of the arXiv, there is a huge amount of work being posted every day. Do not become obsessed with reading everything daily; follow your own interests with integrity, and success will follow.

PH: Thank you very much, Gus. We've had a fascinating discussion of both European history and international mathematics. Your life has been shaped profoundly by both. I wish you the very best for the future.

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Peter Hall was born in Sydney, Australia, and received his BSc degree from the University of Sydney in 1974. His MSc and DPhil degrees are from the Australian National University and the University of Oxford, both in 1976. He taught at the University of Melbourne before taking, in 1978, a position at the Australian National University. In November 2006 he moved back to the University of Melbourne. His research interests range across several topics in statistics and probability theory.