Beyond Turing: Hypercomputation and Quantum Morphogenesis

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1. "Purely Mechanical"

One of the most innovative areas in contemporary research is the study of the deep conceptual connection between physics, information and the "counting" of information, i.e. a search for a computation model for physical systems. Any physical system can be considered as an information processor in dialogue with the external environment. The initial values are transformed into the final ones by the system's internal dynamics. The Church–Turing Thesis (CTT), in its strong form, states that any processing of syntactic information can be described by means of a suitable Turing Machine (TM).

The analogy between a quantum of action and a bit seems very natural (minimum action necessary to cause an observable change in a physical system), and so the CTT, as it is maintained — quite imprecisely — is considered a "statement on the physical world".

Here we ask the question: Is this statement really valid? Is the CT thesis so naturally and obviously applicable to physics?

In recent years the critical debate on the limit of application of recursive functions in physics has grown and, in this direction, the two most important research areas are hypercomputation and quantum information theory.

Some classical works have strengthened the idea that the behaviour of a mechanical discrete system evolving according to local laws is recursive. Such works have shown the relations between the classical computation theory and the physical deterministic systems. In particular, it can be noted a strong analogy between a TM's asymptotic unpredictable behaviours and deterministic chaos; in both cases the local rules do not imply a long-term predictable behaviour, indeed [28, 10, 27, 15]. Different strategies have been proposed to apply such computational scheme to the continuous language of differential equations [22].

The general reasons taken into consideration to justify the use of TM in physics can be summarised as follows:

(a) A fundamental discretisation of the physical world (for example Beckenstein limit: a system cannot handle more information than that it contains);

(b) Relativity: the whole tape is not available in its whole at each computational instant,

and, finally:

(c) The infinity of the tape lets us suppose that there are no limitations to the possible implementations of the Kleene Theorem (for example: asynchronous and parallel computation, cellular automata and so on).

We note, in particular, that the first point refers to a generic discrete structure of the world, but contains no specific information on the proper dynamics of quantum processes, and point (b) is connected to a locality principle. Finally, point (c) stresses the universality of the Turing scheme, with respect to other kinds of computation formally equivalent to the first one, but with a different attitude towards the space-time patterns [40].

In general, the question, if any physical model is Turing computable, collides with a large number of counter examples. These are rather sophisticated questions related to exotic limit-cases of classical, relativistic and gravitational physics (see, for example, [16]), but strong enough to suggest to us that perhaps it would be useful to substitute CT Thesis with a computational paradigm for each specific class of physical problems with the suitable modifications to the (a), (b) and (c) conditions: for example the Friedkin–Toffoli billiard-machine class for mechanical processes, or the class of space time topologies for relativistic computers, or the class of differential equations showing "pathological" boundary conditions with respect to computability.

So the TM remains a notion which was born within a classical and mechanistic conception of the physical world and the Hilbertian axiomatic. As Alan Turing himself writes: "TMs can do anything that could be described as a 'rule of thumb' or 'purely mechanical', so that 'calculable by means of a TM' is the correct accurate rendering of such phrases" [35].

2. A "Paradoxical" Quantum Computing

Quantum computation is not just a technological promise, but the most challenging test for the conceptual problems in quantum mechanics. Nowadays, the Shimony's Experimental Metaphysics is a well-defined activity that is much more complex than any old contraposition between naïve realism and slippery randomness. We could say that Schrödinger's cat has been tamed and is leading us along the most charming paths of the physical world.

It is well-known that (even if all de-coherence problems were solved), quantum computing performances are not qualitatively different from the classical computer ones, except for a few cases when the superposition state makes possible to transform an exponential time of computation in a polynomial time. NP-complete problems appear thus inaccessible even by T-quantum computing. It has been suggested to consider the impossibility to use the known physics' laws to build a computer able to solve NP-complete problems as a new principle — just like thermodynamic ones [1].

A Quantum Turing Machine can be formally defined as an extension of the classical paradigm to qbits (for example, [29]). The results are highly controversial: within such scheme quantum computation does not seem more powerful, but only more effective. And more: in some cases it is possible to demonstrate that the performances of the Turing scheme-based quantum computing can be obtained also by classical systems in polynomial time [2, 7].

All that sounds paradoxical.

In fact, the local and classical world emerges from the non-local quantum one. This one permeates any aspect of the physical world. Turing Machine is a computation model strongly connected to classical, local and deterministic physics. So the proper question is: Is the Turing model really the best scheme for quantum information? The idea of qbit expands easily the traditional concepts of algorithm and universality, but on the other hand the great informational resources of quantum correlations are penalised. In particular, with "quantum gates", the constraints of reversibility and unitarity limit the possibility to detect quantum information only to the outputs of superposition states; and so nothing prevents us from thinking of a different approach to quantum systems, based on peculiar experimental arrangements which can provide qualitatively different answers and oracular skills, so turning into a resource all non-locality features, even those which are traditionally regarded as a limit within the classical scheme, such as de-coherence, dissipation and probabilistic responses.

In other words, Quantum Turing Machines constrain the quantum system to yes/no answers, whereas the real computational vocation of QM would be to use superposition and non-locality to obtain probabilistic oracles and beyond Turing barrier performances. The recent works on adiabatic quantum computation and quantum neural networks [25, 30] thus suggest that a model for the "Schrödinger Machines" has to be searched in a different direction as well as the classical paradigm appears as a "Turing Cage" for the computational potentialities of quantum physics.

Processing information is what all physical systems do. Such intuition first expressed by Rolf Landauer [3] with exemplary clarity has recently risen from the ranks giving birth to extremely interesting and promising developments. The latest computational models have increasingly undermined the privileged position of Turing-Computation model and the role of Church–Turing thesis, as well. The various kinds of unconventional computing focus on either features different from Turing-Machine, such as the greater attention for the spatial and temporal features of computing, or schemes of information processing related to refined forms of non-linearity, fuzziness and infinite or non-computable many values [34].

On the front of biological and cognitive processes, Church–Turing Thesis appears as inadequate, too. Such systems actually show features very far from those required by Turing computability: they are evolutionary, adapting and self-organising, so processes do not "halt", they show intrinsic emergence, are dynamically goal-oriented, modify their codes in relation to the context (semantic appropriation of information), process superposed patterns of continuous information and manage noise. ([21, 8]. For a review on hypercomputation see [33]).

3. There is Plenty of Information at the Quantum Bottom

The quantum case is different in many ways. Bohm's interpretation of quantum phenomena has the merit to include non-locality *ab initio* rather than to come upon it as an *a posteriori* statistical "mysterious weirdness". The Quantum Potential (QP) contains *in* *nuce* the essential features of QM, individualises an infinite set of phase paths for a quantum object and is responsible for entanglement. In particular, the QP has a contextual nature, i.e. it brings a global information — "active information" — on the quantum process and its environment. The active information is defined as a contextual constraint on the phase paths by the quantum potential. It is noted that such interpretation is absolutely general and can be naturally applied to the Feynman path integrals [12, 9, 13].

Here an "active information" field makes its appearance; it has no equivalence in classical physics and indicates the non-local features of quantum domain (superposition and ERP-Bell Correlation). The essential trait of quantum physics is non-locality, which could naturally perform a hypercomputation's feature: exploring "many worlds" in finite (very short) time by means of superposition and entanglement. The active information described is deeply different from the classical one: it is, in fact, intrinsically not-Shannon computable; if it were so, it would mean to violate the Bell Theorem on the impossibility of a QM with local hidden variables. The QC hyper-computational potentialities thus derive from the "unbounded" active information role in acting as "oracular source", in particular experimental configurations. It is possible to think the process as an "entanglement" between the quantum histories of a system, so overcoming the discretisation and locality limits typical of TM. In short, they are what in semi-classical language are called like-space correlations [17, 18].

The problem is how to use such resources. Here the decisive move is to put aside point (c), i.e. the universality of Turing computation and taking into consideration a specific problem-oriented computation and based on its physical implementation.

We proposed the idea of "geometry of effective physical process" as the essentially physical notion of computation [17]. In other words, computation is strongly linked to the very physical nature of the system and its global configuration, and the "algorithm" is the evolution of the system itself in controlled experimental conditions. The notion of geometry also has a significance directly connected to the task: computation — considered as an observer-oriented activity — depends on the adopted experimental configuration, and the hypercomputational potentialities. It is far from being limit situations only occurring in exotic physical environments, depending upon the transformations of the system's geometry. Consequently, the term "programming" takes on a completely new meaning just in relation to the particular geometry the experimental apparatus defines. In the same way as Gödel theorems are considered "limiting" within the Hilbert axiomatic programme. According to Gregory Chaitin vision, they reveal the open logic of mathematics if regarded from a more general viewpoint, the super-Turing possibilities of oracles emerge from a vision which links physics, geometry and information.

By now classical example is the use of the Adiabatic Quantum Computation (AQC) [Farhi *et al.*, 2000; 5, 6, 25, 39, 32], where the problem is solved by following the evolution of a Hamiltonian from a ground state to another according to the constraints suggested by the problem under consideration, so as to give an answer by systematically exploring the Hilbert space. Tien Kieu's work on Hilbert's tenth problem has also suggested interesting thematic hints in constructivist and applied mathematics related to the concept of "proof" and probabilistic answer to a problem. As a matter of fact, the value of the Tien Kieu procedure is "universal" for all Diophantine equations. But it is evident that universality it is here connected to the type of adopted physics, and it is meant differently than under classical computation, because it indicates a dynamical quantum system. During the process, either during elaboration or the final reading, the procedure's quantistic nature is never "forced", hence obtaining the outcomes as probability distributions [26, 24, 23].

Recently the author and his collaborators have investigated a new line of attack to the quantum information problem by using a geometrical approach called Quantum Morphogenetic Computing with reference to the last "analogic" Turing [20, 19]. Starting from a geometric approach to Bohm's quantum potential by Fisher information, we describe the action of a quantum potential by the non-Euclidean deformation in the space of the probabilistic parameters. We use a quantum entropy built as a superposition vector of the Boltzmann entropies. It is rather intuitive to understand that ideally if we "switch off" all the quantum effects, entropy would be to the classical one.

In the condition of minimum Fisher information, the quantum potential emerges as an information channel associated to the deformation of the geometry of the physical space in the presence of quantum effects. In this way it is possible to re-read in a geometric way many traditional quantum phenomena, from the double-slit experiment to the Aharonov–Bohm effect.

With the morphogenetic approach we do a double homage to Einstein's Legacy as for his idea that geometry is at the heart of physics' descriptions [38], and to the Turing of the baby machines and morphogenetic processes.

Finally, we remind, in this respect, that the etymological meaning of the word "information" is, actually, "giving something a new form".

4. Conclusions and Perspectives

The possibility to build a different approach for information is not surprising. The two ways — quantum gates on qbits and author Hamiltonians with constraints — are not in contrast, but complementary. Qbits are more useful when we are interested in individuating a specific state (in Hiley's words: *Shannon information will appear only when we consider a source that could be prepared in one of a number of orthogonal wave functions, each of which could be transferred separately* [14]), and shows a natural vocation for the problems — for instance — typical of nanotechnologies. On the other hand, a geometrical way to handle quantum information is more fruitful when we mean to study the global evolution of a system without forcing its nonlocal nature in any way. The latter way clearly requires new formal tools based on dissipative quantum field theories [31].

Such aspects could be important not only from a technological point of view — for nonlocal communication, cryptography, and exponentially-fast computation — but they could reveal to us something significant on the universe quantum origin and maybe on quantum physics structure itself. It has been hypothesised that the current arrangement of Quantum Theory (Born Rules) is the fossil of a quantum state far from equilibrium with a strong nonlocal correlation and which has played a fundamental role in primeval universe ([36, 37]; see also [11]). Investigating the possibility to reproduce those conditions in laboratory could be the key to extraordinary and completely new resources, and a new pact between physical systems and computation.

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